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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

Edited by

GEORGE E. HALE Mount Wilson Observarory of the Carnegie Institution of Wishington EDWIN B. FROST serkes Observatory of the University of Chicago

HENRY G. GALE Ryerson Physical Laboratory of the University of Chicago

MARCH 1926

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WITH THE COLLABORATION OF

WALTER S. ADAMS, Mount Wilson Observatory JOSEPH S. AMES, Johns Hopkins University ARISTARCH BELOPOLSKY, Observatoire de Poulkows WILLIAM W. CAMPBELL, Lick Observatory HENRY CREW, Northwestern University CHARLES FABRY, Université de Paris ALFRED FOWLER, Imperial College, London

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 itoire de Poulkova
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 CARL RUNGE, Universitä Görtingen

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 SIR ARTHUR SCHUSTER, Twyford

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 Sile Astrona

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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME LXIII

MARCH 1926

NUMBER 2

ABSORPTION SPECTRA OF SOME GASES AND VAPORS IN THE SCHUMANN REGION

By SIGMUND W. LEIFSON

ABSTRACT

The absorption in the Schumann region of oxygen, nitrogen, nitrous oxide, nitric oxide, nitrogen peroxide, carbon monoxide, carbon dioxide, hydrogen chloride, ammonia, methane, water (and vapors of water), carbon tetrachloride, and ethyl alcohol has been studied. A vacuum grating spectrograph was used, with an absorption cell with fluorite windows built into the discharge tube. The continuous spectrum of hydrogen in the region $\lambda 2000-\lambda 1600$ and the secondary spectrum of H from $\lambda 1600$ to $\lambda 1250$ served as the background for the absorption spectra. The fluorite windows transmitted the hydrogen line $\lambda 1215.67$, apparently setting a new ultra-violet limit for the transparency of fluorite.

The results of previous investigators on oxygen, nitrogen, carbon monoxide, and carbon dioxide have been verified and considerably extended. New data have been obtained for the other gases and vapors.

The moment of inertia of the molecule of nitric acid was found to be about 11.2×10^{-40} g cm². Formulae are given for the heads of the bands in terms of frequency for O, CO, CO_2 , and the vapor of ethyl alcohol.

INTRODUCTION

The difficulty in observing absorption spectra in the Schumann region is threefold: first, the production of a source of continuous spectrum; second, the selection of transparent materials for an absorption cell and the rather special technique of ultra-violet spectroscopy that is necessary; and third, the extreme opacity of most substances in this region.

V. Schumann,¹ who was the first to investigate absorption spectra in this region, used a hydrogen discharge tube as a source of radiation; his absorption cell was of fluorite, as well as all the optical

¹ Smithsonian Contributions to Knowledge, 29, No. 1413, 1903.

parts of his vacuum spectrograph, and he used the "Schumann plates" for photographing his spectra. He investigated air and some of the common gases. His results are discussed in the course of this paper in connection with the several gases investigated.

T. Lyman¹ repeated and extended the work begun by Schumann. He used a discharge tube containing a mixture of hydrogen and carbon monoxide as a source of continuous spectrum, a fluorite cell, and a vacuum grating spectrograph.

No other investigators besides Schumann and Lyman seem to have given serious attention to absorption spectra in this region. The present research was attempted for the purpose of extending the work already done in this important field.

APPARATUS AND METHOD

The chief difficulty encountered is due to the lack of a source of light giving a continuous spectrum extending from λ 2000 down to the shortest wave-lengths. E. P. Lewis² showed that hydrogen under certain conditions of excitation emits a strong continuous spectrum extending from the limit of the Balmer series to the beginning of the secondary spectrum near λ 1600. This source of light was found fairly satisfactory.

The discharge tube was of the \cap type with water-cooled electrodes.³ It was made of pyrex tubing 12 mm in diameter. The distance between the electrodes was 100 cm and the length of the horizontal portion 20 cm. While the discharge tube was in operation, dry, electrolytic hydrogen was kept flowing through it by means of a pump connected to the spectrograph. The pressure in the tube could be adjusted to any desired value between 0.1 and 1 mm, by means of a needle valve made of glass and operated with an electromagnet. The current was obtained from a 10-k.-v., 6600-volt transformer. Neither condenser nor spark gap was used in the secondary circuit. It was found that with a pressure in the tube of 0.5 mm and a current of 0.3 amp. an intense continuous spectrum was produced. The time of exposure varied from forty-five to ninety minutes.

The concave-grating vacuum spectrograph described by Hop-

¹ Astrophysical Journal, 27, 87, 1908. ² Science, N.S., 41, 947, 1915.

³ J. J. Hopfield, Astrophysical Journal, 59, 114, 1924.

ABSORPTION SPECTRA OF GASES

field^{**i**} was used. The films were prepared in the manner described by Hopfield but with only one-half the amount of gelatin called for in the formula for the Schumann emulsion. These seemed to be much faster than those made according to the regular process.



FIG. I



FIG. 2

The absorption cell, shown in Figures 1 and 2, was also made of pyrex, and it was sealed to the discharge tube. The fluorite windows were sealed on with red wax. The distance between the windows was 5, 15, and 25 mm, respectively, in the different absorption cells used. The cell was provided with a side tube containing a ground joint in

¹ J. J. Hopfield, Physical Review, 20, 573, 1922.

order to give access to the interior for cleaning or changing the windows. This side tube also served as container for liquids whose vapors were to be examined. The pressure of the vapor was determined by the temperature of the liquid. The temperature was kept constant by means of a Dewar flask. Some of the auxiliary apparatus connected with the absorption cell is shown in Figure 2. The gas, whose absorption spectrum was to be studied, was prepared, purified, and dried, and then passed into the absorption cell. A steady flow of the gas through the cell was maintained by means of a pump. The two needle valves shown in the figure made it possible to keep the pressure in the cell constant at any value between 5 and 750 mm. The pressure gauge consisted of a combined manometer and McLeod gauge.

Much difficulty was encountered with the fluorite windows. After thirty to forty hours' exposure they became quite opaque to all light beyond the near ultra-violet. This effect was rather gradual, and it was probably due to the formation of opaque surface films. The windows have not been repolished to determine this point with certainty.

Some of the spectrograms obtained are shown in Plates IV-V. They were secured in the following way. The absorption cell was evacuated and filled with dry hydrogen several times. Then an exposure was made with dry hydrogen at a pressure of 10 mm in the cell. The spectrum thus obtained is shown at the top in each plate. It shows the nature of the background and serves as a comparison for the absorption spectra below it. Next, three to five exposures were made on the same film with a given gas in the absorption cell at pressures ranging from 0.01 and 1 atmosphere.

The ultra-violet limit of the spectrum at λ 1240 is due to the absorption of fluorite. This limit, however, is not as sharp as the results of previous investigators indicate because the first line of the Lyman series, λ 1215.67, appears on most of the spectrograms.

The wave-lengths of absorption bands were measured relative to strong lines in the secondary spectrum of hydrogen and to the nitrogen doublet, λ 1742.81 and λ 1745.31, which occurs in this region. The wave-lengths of the hydrogen lines were obtained from one of Dr. Hopfield's spectrograms containing hydrogen and air





PLATE IV

a, Oxygen. Thickness of absorption cell, 25 mm. Pressure in absorption cell: A, 18 mm; B, 240 mm; C, 740 mm.

b, Nitrogen. Thickness of cell, 5 mm. Pressure in cell: D, 150 mm; E, 750 mm.

c, Nitric oxide. Thickness of cell, 15 mm. Pressure in cell: F, 7 mm; G, 75 mm; H, 750 mm.

ABSORPTION SPECTRA OF GASES

lines. The latter are known to 0.1 A.^I In the case of oxygen and nitric oxide, some of the bands could be measured in the second order with lines of the Balmer series as standards. The given wave-lengths are in I.A. *in vacuo*, and they refer to the centers of the unresolved bands. In most cases the accuracy is probably not greater than I A, although the relative accuracy is greater in the case of the oxygen bands.

The computation of series formulae was greatly facilitated by means of the directions contained in "A Rapid Method of Calculating the Least Squares' Solution of a Polynomial of Any Degree," by R. T. Birge and J. D. Shea.⁴

RESULTS

Oxygen.—Commercial oxygen dried with phosphorus pentoxide was used. The absorption cell had a thickness of 25 mm.

The absorption of oxygen has been studied before by Schumann,³ Lyman,⁴ and J. Ducleaux and P. Jeantet.⁵ Schumann found a group of fourteen bands shaded off toward the red near λ 1850. He was not able to determine the wave-lengths of the heads of the bands, and he did not observe any fine structure. Lyman discovered a continuous region of absorption in oxygen extending from λ 1720 to λ 1300, but he was not able to find the selective absorption mentioned by Schumann. Hopfield⁶ found that the λ 1300 transmission band ends and an absorption band begins again at λ 1000. Ducleaux and Jeantet studied the absorption of oxygen between λ 2000 and λ 1900. Their work will be mentioned later.

The Schumann bands were observed and their fine structure was also obtained by the author (see Plate IV). The heads of the bands are expressed fairly accurately by the relation:

$\nu = 56,931 - 76.63m - 20.056m^2$.

Table I gives the observed values and values calculated by this formula. In addition to those listed, there were two more bands at

¹ J. J. Hopfield and S. W. Leifson, Astrophysical Journal, 58, 59, 1923.

² Physical Review, 24, 206, 1924. (Abstract.)

³ Loc. cit. ⁴ Loc. cit. ⁵ Comptes Rendus, 173, 581, 1921.

⁶ J. J. Hopfield, Physical Review, 20, 573, 1922.

 λ 1948 and λ 1970 too faint to be accurately measured. Table II gives the fine structure of four of the bands. In the rest of the bands it was distinct enough to be seen, but not sufficiently sharp to be measured with any degree of accuracy. Some of these bands have been obtained by indirect methods by W. Steubing¹ and by L. and E. Bloch.² A comparison with their values is shown in Table III. Steubing believed that the bands obtained by him were emission bands, and he attributed them to fluorescence of oxygen. This con-

TABLE	I
OXYGEN-BAND	HEADS

295	λ Obs.	v Obs.	» Calc.	Diff.
0	1756.7	56,925	56,931	- 6
I	1759.6	56,831	56,834	- 3
2	1763.8	56,696	56,698	- 2
3	1769.2	56,523	56,521	+ 2
4	1775.9	56,309	56,304	+ 5
5	1783.9	56,056	56,046	+10
6	1793.4	55,760	55,749	+11
7	1804.3	55,423	55,412	+11
8	1816.8	55,042	55,034	+ 8
9	1831.1	54,612	54,617	- 5
10	1846.9	54,145	54,159	-14
II	1864.2	53,642	53,663	-21
12	1883.0	53,107	53,123	-16
13	1903.I	52,546	52,545	+ 1
14	1924.8	51,954	51,927	+27

clusion has been questioned very much, and L. and E. Bloch do not hesitate to attribute the bands to absorption. They obtained the same bands with a spark discharge between various metallic electrodes in air.

Ducleaux and Jeantet³ used an absorption cell of 50-cm thickness, and they found four bands between $\lambda 2000$ and $\lambda 1900$. They designated these bands as belonging to group II, and showed that they were similar in nature to the Schumann bands which they designated by group I. They found that there was a constant frequency-difference between corresponding bands of the two groups. Their data are shown in Table IV.

- ¹ Annalen der Physik, 33, 533, 1910.
- ² Comples Rendus, 158, 1161, 1914.

3 Ibid., 173, 581, 1921.

TABLE II

FINE STRUCTURE OF OXYGEN BANDS

 $v = 55.037 - 31.97m - 2.047m^2$

_	m										_	_							₽ Obs.	» Calc.	Diff.
0																			55,042	55,037	+5
1						*						•	*		*				54,998	55,003	-5
2.						*											,		54,962	54,964	-2
3.												•							54,925	54,924	+1
4.						•								•					54,878	54,876	+2
5.				*				• •						*					54,830	54,826	+4
6.							•								•			.	54,771	54,771	0
7.									. ,				*		*	•			54,712	54,713	-1

 $\nu = 54,606 - 31.86m - 1.833m^2$

_	993															v Obs.	▶ Calc.	Diff.			
0			•														0		54,612	54,606	+6
1			•	•		•	•									•	•	0	54,566	54,572	-6
2		• •		•	•		•		•	•									54,532	54,535	-3
3					•			•		•		•		•					54,495	54,494	+1
4			•					•						•	•		•		54,452	54,449	+3
5							*		*		*		*	*		*			54,404	54,401	+3
6,		.,			*							*				*			54,350	54,349	+1
7.				•			•				*								54,291	54,293	-2

 $\nu = 54, 141 - 29.51m - 2.070m^2$

_	111									_			_		_	_	v Obs.	» Calc.	Diff.	
0.				•	•								•		•			54,145	54,141	+4
I.			•	•		• •						*						54,105	54,109	-4
2.	•			•												•		54,071	54,074	-3
3.			•			• •												54,032	54,034	-2
4.		•			•													53,990	53,990	0
5.					• •					*	•	*						53,943	53,941	+2
5.		*																53,891	53,889	+2
7.		*																53,833	53,832	+1
8.																		53,769	53,772	-3

 $\nu = 53,638 - 28.36m - 2.027m^2$

_	m									_						v Obs.	v Calc.	Diff.	
0.					•								•	•			53,642	53,638	+4
I.					•	*									*	*	53,602	53,608	-6
2.					•								•	*			53,570	53,573	-3
3.		•		•	*	•									•	•	53,533	53,535	-2
4.	•				*	•								*			53,493	53,492	+1
5.					*								*		*		53,447	53,446	+1
6.					•					0	0	۰	•				53,396	53,395	+1
7.	*				*						*					•	53,339	53,340	-1
8.					•										•	•	53,279	53,281	-2

These bands were not obtained in the present work on account of the small thickness of oxygen used. Corresponding bands of the

CT3 4	TAT	100		-
- A	ы	. H.:		
4.4.3	1.1.1	4.8.4	**	ж.

PRESENT WORK	STEUBING	L. AND E. BLOCH			
v Obs.	» Obs.	۶ Obs.			
3,642	53,648	53,670			
3,107	53,135	53,139			
52,546	52,631	52,578			
1,954		51,987			

TA	BI	LE	I	V
A 6 8	and a	110		

Group I	Group II	Difference $\Delta \nu$
3,670	52,116	1554
3,139	51,585	1554
2,578	51,021	1557

two groups have the same structure. Ducleaux and Jeantet give the following formulae as representing two of the bands:

$$v = 52,616 - 3.1m^2 \ (m = 1, 2, 3 \dots 10)$$

$$v = 52,116 - 3.1m^2 \ (m = 1, 2, 3 \dots 10).$$

The corresponding bands of group I found in the present work may be approximately represented by

> $\nu = 54,145 - 3.1m^2 \ (m = 4,5,6 \ \dots \ 11)$ $\nu = 53,642 - 3.0m^2 \ (m = 4,5,6 \ \dots \ 11).$

Thus there appears to be a close connection between the bands of the two groups. This point will be discussed further in a later paragraph.

Nitrogen.—Commercial nitrogen was used. It was purified by passage through hot copper dust, and it was dried with phosphorous pentoxide. The absorption cell had a thickness of 5 mm.

Schumann¹ found selective absorption in nitrogen. Lyman² ¹ Loc. cit. ² Loc. cit.

ABSORPTION SPECTRA OF GASES

found no selective absorption, only a very weak general absorption increasing in intensity with decreasing wave-length.

The present work shows that the absorption of nitrogen in the Schumann region is in the form of a band with a maximum at λ 1440 (see Plate IV). With 5 mm of nitrogen at atmospheric pressure, this absorption band extends from λ 1520 to λ 1370. When the pressure of the gas in the absorption cell is reduced to one-fifth of an atmosphere, the absorption is no longer noticeable. An experiment was also made with nitrogen obtained chemically from ammonium nitrite, and there were found a number of narrow bands which were subsequently identified with similar bands in nitric oxide. It is very probable that Schumann used chemically prepared nitrogen. In that case it is also very probable that the selective absorption, which he observed, was due to nitric oxide.

Nitrous oxide.—This gas was prepared by heating ammonium nitrate and dried with phosphorus pentoxide. The absorption cell had a thickness of 15 mm.

Nitrous oxide shows no selective absorption in the Schumann region. With the gas in the cell at atmospheric pressure, the absorption is in the form of two continuous bands, the first extending from λ 2000 to λ 1680, and the second from λ 1550 beyond the range of observation.

Nitric oxide.—This gas was prepared by dropping concentrated nitric acid into a boiling solution of ferrous sulphate and dilute sulphuric acid. It was dried with phosphorus pentoxide. The absorption cell had a thickness of 15 mm.

Exposures were made with nitric oxide in the absorption cell at pressures of 0.01, 0.1, and 1 atmosphere. The spectra obtained show a large number of narrow bands (see Plate IV). Those on the side toward red of λ 1800 are double. There was no observable continuous absorption. The wave-lengths in Table V indicate the centers of the bands. These bands seem to form no regular series. A number of similar groups were noted, as shown in Table VI.

Nitrogen peroxide.—This gas was prepared with copper and concentrated nitric acid and dried with phosphorus pentoxide. The absorption cell had a thickness of 15 mm.

Of all the gases examined in this investigation nitrogen peroxide was found to be the most opaque. No evidence of selective absorption in the Schumann region was obtained. With nitrogen peroxide in the absorption cell at atmospheric pressure all light below λ 4000 was absorbed. As the pressure in the cell was reduced to 150 and 30 mm, total absorption occurred at λ 2400 and λ 2100, respectively.

λ	v	λ	p
2264.7	44,156	1642.3	60,890
2150.9	46,492	1621.1	61,686
2111.1	47,369	1610.6	62,089
2067.2	48,375	1605.6	62,282
2048.8	48,809	1599.5	62,519
2026.3	49,351	1593.8	62,743
1986.7	50,335	1581.1	63,247
1957.4	51,088	1556.8	64,234
1950.4	51,272	1552.1	64,429
1910.6	52,340	1527.7	65,458
1877.2	53,271	1502.2	66,569
1847.4	54,130	1497.2	66,791
1829.2	54,669	1492.2	67,015
1799.6	55,568	1469.1	68,069
1789.4	55,885	1452.6	68,842
1764.0	56,689	1448.0	69,061
1753.3	57,035	1443.5	69,276
1740.0	57,471	1423.0	70,270
1729.7	57,813	1407.8	71,033
1709.4	58,500	1390.9	71,896
1690.1	59,168	1376.6	72,643
1677.7	59,605	1362.9	73,373
1666.6	60,000	1296.4	77,137

	TA	BLE V	
BANDS	IN	NITRIC	OXIDE

When the pressure was reduced to 6 mm, two broad absorption bands appeared with maxima at λ 1700 and at λ 1400, and also a third band extending from λ 1325 beyond the range of observation.

Carbon monoxide.—This gas was produced by heating a mixture of oxalic and sulphuric acids. It was passed through a strong solution of potassium hydroxide to remove carbon dioxide and then dried over phosphorus pentoxide. The absorption cell had a thickness of 15 mm.





PLATE V

a, Carbon monoxide. Thickness of absorption cell, 15 mm. Pressure in absorption cell: A, 75 mm; B, 750 mm.

b, Ammonia. Thickness of cell, 15 mm. Pressure in cell: C, 3 mm; D, 100 mm; E, 730 mm.

c, Water and water-vapor: F, thin film of water; G and H, water-vapor. Thickness of cell, 15 mm. Pressure in cell: G, 20 mm; H, 5 mm.

The absorption of carbon monoxide in the Schumann region has been investigated by Schumann¹ and by Lyman.² Schumann observed a series of "rhythmical, inverted groups of lines." Lyman found eight narrow bands, of which wave-lengths are given in Table VII for comparison.

The selective absorption mentioned by Schumann was not found in this investigation. There was, however, a faint band of uncertain

	I	
λ	P	$\Delta \nu$
2264.7		
2150.9		2336
2048.8		2317
1957.4		2279
	II	
2111.1		
2067.2		1006
2026.3		976
1986.7		984
1950.4		932
	III	
1877.2		
1799.6		2297
1729.7		2245
1666.6		2187

TABLE VI							
BAND	GROUPS	IN	NITRIC	OXIDE			

character between λ 1820 and λ 1760. It is probable that with a longer absorbing column the lines observed by Schumann would appear in this region, for his observations did not extend beyond λ 1600.

A group of twelve narrow bands, eight of which were observed by Lyman, was found (see Plate V). They seem to continue beyond the limit of transparency of fluorite. Their wave-lengths are given in Table VII. The present work shows that the wave-numbers of these bands are expressed within the limit of experimental error by the relation:

 $\nu = 64,678 + 1487.5m - 15.92m^2$.

Loc. cit.

* Loc. cit.

Carbon dioxide.—This gas was prepared by hydrochloric acid and calcium carbonate, washed with sodium bicarbonate, and dried with phosphorus pentoxide. The absorption cell had a thickness of 15mm.

The absorption of carbon dioxide in the Schumann region has been previously investigated by Schumann and by Lyman. Schumann found absorption bands similar to those of oxygen, but of much shorter wave-length. Lyman found "indications of maxima and minima of absorption in the region beyond λ 1850."

With carbon dioxide in the cell at atmospheric pressure there was no noticeable absorption above λ_{1712} . At this point, absorption

PT A	DT	TP2	3.7	TT	
IA	BL	L.	- V .	11	

BANDS	IN	CARBON	MONOXIDE
-------	----	--------	----------

936	λ Obs.	Lyman	v Obs.	v Calc.	Diff.
0	1545.6	1548	64,700	64,678	+22
I	1511.7	1512	66,148	66,149	- I
2	1480.2	1482	67,558	67,589	-39
3	1450.3	1450	68,951	68,997	-46
4	1421.9	1423	70,328	70,373	-45
5	1394.7	1395	71,697	71,717	- 20
6	1369.0	1370	73,046	73,030	+16
7	1345.2	1345	74,333	74,310	+23
8	1323.2		75,570	75,559	+11
9	1302.3		76,782	76,776	+ 6
0	1283.0		77,942	77,961	-19
1	1264.6		79,076	79,114	-38

was found to begin in the form of bands shaded off toward the red. These are presumably the bands mentioned by Schumann and by Lyman. There is also considerable general absorption which makes the bands indistinct. Total absorption begins at λ 1610. When the pressure in the absorption cell is reduced to one-fifth of an atmosphere, the selective absorption appears to extend from λ 1600 to the limit of transparency of fluorite. On account of the many strong lines in the secondary spectrum of hydrogen occurring in this region, only a few of the absorption bands of carbon dioxide could be measured. The wave-lengths of the heads of the bands are given in Table VIII. The wave-numbers of the bands' heads are given by the relation:

 $\nu = 59,102 + 630m - 31m^2$.

With carbon dioxide in the absorption cell under a pressure of 30 mm, there was no noticeable absorption.

Hydrogen chloride.—This gas was obtained by heating concentrated hydrochloric acid. It was dried with phosphorus pentoxide. The cell had a thickness of 15 mm.

With hydrogen chloride in the cell at atmospheric pressure, all light below λ 2350 was absorbed. When the pressure was reduced to one-fifth of an atmosphere, total absorption occurred at λ 2250; and at one-fiftieth of an atmosphere, four broad, continuous absorption bands appeared. The approximate limits of these bands are

TA	BL	Æ	V	III	

BANDS IN CARBON DIOXIDE

995	λ Obs.	» Obs.	v Calc.	Diff.
0	1692	59,102	59,102	0
I	1675	59,701	59,701	0
2	1660	60,241	60,238	+3
3	1047	00,710	00,713	+3

 λ 2150- λ 1850, λ 1750- λ 1650, λ 1580- λ 1290, and from λ 1270 beyond the range of observation. A very intense, narrow band appeared at λ 1289 when the pressure was reduced to 10 mm. At 1-mm pressure, this band was reduced to a narrow line. The change in width was symmetrical.

Ammonia.—This gas was obtained by heating concentrated ammonium hydroxide, and it was dried with calcium oxide. The absorption cell had a thickness of 15 mm.

With ammonia in the absorption cell at atmospheric pressure, there appeared three absorption bands at λ 2260, λ 2210, and λ 2166 (see Plate V). Beyond the last band the absorption was complete. These are probably the same as those observed by Ferrières.^r The bands increase rapidly in width with decreasing wave-length. It was therefore inferred that the apparent general absorption was really due to a long series of bands, and this inference was subsequently verified by means of an exposure with the ammonia in the absorp-

¹Science Abstracts, 27, No. 1172, 1924. Only the abstract was available, and it did not give sufficient information.

tion cell at a pressure of about 3 mm. The spectrum thus obtained shows a group of regularly spaced bands extending from λ 2210 to λ 1515. Because of the diffuse character of the bands, their wavelengths could not be determined very accurately. The approximate wave-lengths of the centers of the bands are given in Table IX. The bands extend across the gap from λ 1813.7 to λ 1614, but they were too faint to be measured. The frequency differences appeared to in-

TABLE IX

DANDS IN AMMONIA	
λ ν	$\Delta \nu$
2260	
2210.1	999
2166.5	910
2127.5	847
2089.0	866
2050.5	899
2012.6	918
978.0	869
044.0	884
	878
877.4	947
844.8	94I
813.755,136	930
614	
589	975
564 1	006
539 1	038
515 1	029

crease slowly with decreasing wave-length. No attempt was made to obtain an equation for the wave-numbers of these bands, because of the irregularities in the successive differences. It seems probable, however, that the correct wave-numbers could be accurately expressed by a polynomial of the second degree. The bands appear to decrease uniformly in intensity from the middle of the group toward both sides.

Methane.—The gas was prepared by distillation of a dry mixture of sodium acetate and soda lime. It was dried with phosphorus pentoxide. The cell had a thickness of 15 mm.

With methane in the absorption cell under a pressure of I

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atmosphere, all light beyond λ 1800 was absorbed. As the pressure in the cell was reduced, this limit receded toward the violet. At the lowest pressure used, 6 mm, there appeared a number of bands with centers at λ 1558, 1528, 1501, 1474, 1448, and 1420, approximately. No attempt was made to calculate a formula for this group of bands because of uncertainty in the measurements of the wave-length.

Water and water-vapor.—A thin film of water due to the condensation of water-vapor on the windows of the cell absorbed all light beyond λ 1790. Lyman¹ found 0.5 mm of water transparent to λ 1792. H.Kreusler² observed 68.9 per cent absorption with 16.97 mm of water at λ 1860. By extrapolation of his results, λ 1830 is found as the limit of transparency of 16.97 mm of water.

An absorption cell of 15 mm thickness was used for water-vapor. Previous investigators have failed to obtain definite results on the absorption of water-vapor in the Schumann region. The results of the present investigation show (see Plate V) that water-vapor has a strong absorption band between λ 1780 and λ 1610 with a maximum at λ 1700. At λ 1392 a second absorption band begins; its limit on the side of shorter wave-lengths is beyond the range of observation.

Carbon tetrachloride vapor.—Pure carbon tetrachloride was used. The cell had a thickness of 25 mm.

Carbon tetrachloride vapor shows no selective absorption. With a vapor pressure of 90 mm in the absorption cell, total absorption begins at λ 2100. As the pressure is reduced, the limit of transparency moves slowly toward the ultra-violet. With a vapor pressure of 10 mm, the absorption is in the form of two bands. The first extends from λ 1840 to λ 1670; the second begins at λ 1530 and extends beyond the range of observation.

Ethyl alcohol vapor.—Pure ethyl alcohol was used. The absorption cell had a thickness of 25 mm.

With this vapor at a pressure of 6 mm in the cell, total absorption occurs at λ 1570. On the side of this limit toward longer wavelengths there appear a number of equally spaced, narrow bands whose wave-numbers are given by the relation:

v = 49,044 + 875m.

1 Nature, 84, 71, 1910.

² Annalen der Physik, 6, 418, 1901.

The bands are listed in Table X. Those at λ_{1969} and λ_{1936} are most intense. The intensity decreases rapidly toward both sides of the group. It is therefore probable that under more favorable conditions additional bands might be obtained with wave-lengths longer than λ_{2039} and others shorter than λ_{1872} .

T	A D'	F TP	v
	AD.	LL	~

BANDS IN ETHYL	ALCOHOL	VAPOR
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_		1	98		λ Obs.	v Obs.	» Calc.	Diff.
0					2039	49,044	49,044	0
I.					2003	49,925	49,919	+ 6
2					1969	50,787	50,794	7
3				 0	1936	51,652	51,669	-17
4.		•		 	1902	52,576	52,544	+32
5.					1872	53,419	53,419	0

CONCLUSION

The absorption spectra of molecules consist of narrow and broad bands. The narrow bands, as a rule, contain fine structures which can be studied only with spectrographs of high dispersion. The general theory of band spectra applies to such bands. The broad bands are, in some cases at least, really continuous. There is no satisfactory theory to account for them. According to the quantum theory of band spectra, a group of bands in the ultra-violet is due to molecules undergoing the same electronic change, but different vibrational changes. Each band is due to a particular type of vibrational change. The fine structure arises from the various types of rotational changes. The interval between successive components at the origin of the band in frequency units ($\Delta \nu_0$) gives the moment of inertia of the radiating molecule according to the formula:

$$J=\frac{h}{4\pi^2\Delta\nu_0},$$

where h is Planck's constant. On account of the fact that the rotational energy may change by -1, o, or +1, it should be possible to arrange the components of the bands into three branches, called the P, Q, and R branches, respectively. In the case of molecules that have no electrical moment, such as O_2 and N_2 , however, the Q

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branch does not occur. If, therefore, such a band is examined with an instrument of insufficient resolving power, it will appear as a double band. According to E. C. Kemble,^x the distance between the maxima of the two branches should be given by the expression:

$$\Delta \nu = \frac{\mathrm{I}}{\pi} \sqrt{\frac{RT}{NJ}} \, .$$

In the case of the oxygen bands given on page 78, the origin of each was presumably too close to the head to be found. Therefore, the moment of inertia of the molecule could not be computed.

The groups of bands, I and II, given on page 80, probably form two sequences, each of which is characterized by a constant difference in the vibrational quantum numbers of the initial and final states. According to the quantum theory of band spectra, the constant frequency-difference between corresponding bands of the two groups should give the actual frequency of vibration of the dipole, which in this case would be of the right order of magnitude.

It is possible to obtain an approximate value for the moment of inertia of the nitric oxide molecule from the double bands found in this gas. It is assumed that in each branch of a double band the intensity rises sharply to a maximum. The separations of the maxima of four successive bands were 63, 68, 64, and 63 in units of wavenumber. The average separation was 64.5. The corresponding moment of inertia is about 11.2×10^{-40} g cm.² This value is of the right order of magnitude.

In connection with this work the author is much indebted to Dr. J. J. Hopfield, who furnished the use of his splendidly adjusted concave-grating vacuum spectrograph, and who followed the progress of the work with constant interest and valuable advice. The author also wishes to express his gratitude to Professor R. T. Birge for helpful suggestions on the theory of band spectra.

DEPARTMENT OF PHYSICS UNIVERSITY OF CALIFORNIA May 1925

* Physical Review, 8, 689, 1916.

PHYSICAL THEORY OF METEORS

By C. M. SPARROW

ABSTRACT

Heating of a meteor by molecular impacts.—When the free path of air molecules is not small compared to the dimensions of the meteor, the ideas of *fluid motion* are inapplicable, and the "friction" of the air is seen as a succession of individual molecular *impacts*. Assuming this input of energy to be balanced by radiation losses, the temperature may be calculated when the speed and the composition of the air are known.

Height of appearance.—When the evaporation of the solid body becomes copious, the accession of energy is much increased because of the greater effective size of the gas mantle. The meteor then becomes visible. The theory is shown to give results in good agreement with observed heights of appearance and with the data concerning the upper air which are furnished by the meteorologists.

Criticism of theory of Lindemann and Dobson.—The theory of these writers rests on the assumption of adiabatic compression of the air by the moving meteor. It leads to the assumption of pressures and temperatures in the upper air far higher than have been generally accepted. Their use of thermodynamic concepts is here seen to be unjustifiable, and hence their conclusions unwarranted.

Origin of Meteors.—That a large proportion of meteors come from outside the solar system is contrary to this theory, the hyperbolic velocities giving a height of appearance much greater than that of the Leonids, and such are relatively rare.

In the following paper an attempt is made to present a connected account, in physical terms, of what happens when a solid body enters the upper atmosphere of the earth at high speed. Such bodies may differ in composition, size, shape, and in their speed and the inclination of their path to the vertical. If to this list we add the composition and physical state of the upper air, we have enumerated all the independent variables at our disposal. In terms of these we must express the heights of appearance and disappearance, the kind and amount of radiated energy, and a number of minor peculiarities which the phenomena exhibit. The theory here offered can make no pretensions to affording a complete answer to all these questions, but it appears to account for all of them at least qualitatively, while showing good quantitative agreement with observation on those points at which such a test is possible. Since it has further the advantage of requiring no extensive modification of our current notions (particularly regarding the upper air), it is hoped that it will commend itself to astronomers and physicists as a basis for further work.

Before entering upon the development of the theory itself, we

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must clear the ground by a discussion of a few facts and data which enter into it. Of these the first in importance relate to the upper air. Table I, taken from W. J. Humphreys' *Physics of the Air*, may be regarded as a summary of the prevailing opinion of the meteorologists on this subject. The figures give the percentage composition and total pressure.

HEIGHT IN	Gases							
КМ	A	Na	H ₁ O	01	CO.	H ₂	He	PRESSURE IN MM
140		0.01				99.15	0.84	0.0040
130		.04				99.00	0.96	.0046
120		. 19				98.74	1.07	.0052
110		0.67	0.02	0.02		98.10	I.19	.0050
100		2.95	.05	.II		95.58	I.31	.0067
90		9.78	. 10	0.49		88.28	I.35	.0081
80		32.18	.17	1.85		64.70	I.10	.OI23
70	0.03	61.83	. 20	4.72		32.61	0.61	.0274
60	.03	81.22	.15	7.69		10.68	.23	.0035
50	.12	86.78	. 10	10.17		2.76	.07	0.403
40	. 22	86.42	.06	12.61		0.67	.02	1.84
30	.35	84.26	.03	15.18	0.0I	. 16	0.01	8.63
20	.59	81.24	.02	18.10	.0I	.04		40.00
15	.77	79.52	.OI	19.66	.02	.02		89.66
II	.04	78.02	.01	20.00	.03	01		168.00
5	.04	77.80	0.18	20.95	.03	.01		405.
0	0.03	77.08	I.20	20.75	0.03	O.CI		760.

	TA	BL	ΕI			
PERCENTAGE	DISTRIBUTION	OF	GASES	IN	THE	ATMOSPHERE

Our direct knowledge of the upper air extends only to about 30 km. For greater heights, the figures of Table I are an extrapolation based on the following assumptions: up to 11 km there is convective equilibrium, with mixing and consequent uniformity of composition (save for the diminishing water-vapor), and a temperature which falls uniformly with the height to the value 219° K. at 11 km. Above this point the temperature is assumed constant; there is no mixing, each constituent being distributed independently of the others, so that the whole may be regarded as a system of independent and interpenetrating atmospheres. The table being based on mean values allows a certain range of variation on account of seasonal and local conditions. Thus, at the equator, the tabulated value for 140 km would correspond to about 145 km.

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The extrapolation from 30 to 140 km, involving an assumed diminution of pressure to about 1/2000 of the last experimental value, appears at first sight to be a very tenuous thread on which to hang any quantitative reasoning. It must be remembered, however, that this extrapolation is not empirical, but is based on a theory which has been verified between 11 and 30 km. On one point only have the data up to this height been seriously questioned; this point concerns the amount of hydrogen. The value given, I part by volume in 10,000 at sea-level, is that usually adopted; but Lord Rayleigh finds the amount to be less than I part in 30.000. Claude less than I in 35,000, while some deny its presence altogether. As the theory to be given throws some light on the question, we will defer a more detailed discussion," but one point should be made clear at once. If we accept the view that there is no convection at high altitudes, the total amount of other gases present will be unaffected by the presence or absence of the hydrogen. Thus, if it be assumed altogether lacking, we should have, on the hypothesis underlying the table, an atmosphere which at 140 km would consist of about 1.2 per cent nitrogen and 98.8 per cent helium, with a total pressure of about 3×10^{-5} mm. If the helium also is denied, there would be practically pure nitrogen, but its total pressure would be only 4×10^{-7} mm. It is practically certain that meteors can appear as high as 160 km, at which the pressure would fall to about 1/25 of this value. Any substantial increase in the amount of nitrogen at these heights involves the assumption of higher temperatures. Though the assumptions of the meteorologist are intrinsically reasonable, the right to challenge them on the basis of other evidence cannot be denied. We are not, however, at liberty to play fast and loose, and to assign temperatures and pressures to suit our fancy; for the gas law and the hydrostatic equation impose limitations which we may not abrogate.

To see how the matter stands, we start from the two equations

$$p = R\rho T; \qquad \frac{dp}{dh} = -\rho g,$$

* For detailed references see Jeans, Dynamical Theory of Gases (3d ed.), p. 340 n.

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from which, eliminating ρ , we get

$$\frac{dp}{dh} = -\frac{gp}{RT} \text{ or } \log \frac{p}{p_0} = -\frac{g}{R} \int_{h_0}^{h} \frac{dh}{T}.$$

If T is given as a function of h, the pressure at any height is determinate. As T is essentially positive, the absolute value of the integral on the right is greater than $(h-h_0)/T_{max}$, and hence

$$\frac{p}{p_o} \le e^{-\frac{g(h-h_o)}{RT_{max}}},$$

or, in other words, the actual pressure under a variable temperature will be less than that calculated on the assumption of an isothermal distribution at the maximum temperature existing below it. Since for constant T the density varies as the pressure, the density will satisfy the same inequality.

The form of the equation lends itself to easy orientation among the figures without extensive calculation, for we see that we can use the figures of Humphreys' table simply by changing the scale of heights proportionately to the absolute temperature. Assume, for instance, that while the tabulated values are correct up to 30 km, above this height the temperature changes so that the pressures and densities which it gives for 110 km actually exist at 150 km. The maximum temperature will then be at least as great as $219 \times (150-30)/(110-30)$, or 328° K.

A theory of meteors was put forward a few years ago by Lindemann and Dobson¹ in which they reach conclusions regarding the density of nitrogen at high altitudes that are much at variance with current assumptions. Some of the reasoning on which these conclusions are based will be criticized later, but the writer finds himself unable to verify even the consistency of those conclusions. They assume, apparently, a temperature of about 300° K. The exact figures for their assumed density are not explicitly given. If the writer understands rightly the diagram on page 427 of their paper cited above, the value 219° K. is assumed to hold for T up to 40 or

¹ Proceedings of the Royal Society of London (A), 102, 411, 1921-22.

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50 km and the density at 150 km is assumed about equal to the accepted value at 90 km. This, however, would give a temperature of at least $219 \times (150 - 40)/(90 - 40)$, or 480° K. To turn the calculation the other way: a temperature of 300° K. would permit at 150 km a density not greater than that tabulated for 120 km; an increase by a factor of about 90, instead of the factor of 1000 which they give on page 428. Lindemann and Dobson appear either to have made an error in their calculations, or to have stated their conclusions in a form susceptible of misinterpretation.

This rather lengthy discussion of the atmospheric problem seems justified by its fundamental importance, and by the doubt which has been cast on the views hitherto prevailing. We assume in the calculations to follow, the substantial correctness of Humphreys' table, with a possible exception as to the amount of hydrogen, for which it is felt that a reduction of the assumed amount would be amply justified by existing evidence, should the theory seem to require it. The table in its present form does not lend itself, however, to easy use in calculation, and is more conveniently replaced by Table II, which gives the ratio of the density at a given height to the standard density. The minor constituents have been omitted, and the range of heights is extended to 200 km.

The relevant facts regarding meteors themselves may be more briefly considered. The composition varies considerably, the siderites being almost pure iron, while the aerolites show a great variety of minerals similar, but not always identical with, those constituting terrestrial rocks. The smaller the meteor, the greater is the presumption of homogeneity, but the particular mineral composing it is unknown to us, and the writer has not been able to find any data bearing on the behavior of meteoric minerals at high temperatures. It would be very convenient if the iron type could be assumed as the usual thing; but the statistics of recorded falls indicate the relative rarity of this type. The data to be used will be introduced as needed; as these values are largely guesses, the usual procedure will be to attempt an estimate of upper and lower limits, and to make alternative calculations.

The size of meteors may of course vary enormously. A lower limit is obtained by equating the total radiated energy to the original

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kinetic energy of the particle. On the plausible view that the actual size is not a large multiple of this it has been concluded that the diameter of an ordinary shooting star, of magnitude o or fainter, does not exceed a few millimeters. We shall attempt later a more precise estimate, but this rough guess is sufficient for present purposes.

The speed is usually calculated on the assumption of a parabolic orbit about the sun, which gives a possible range of speeds from 16 to 72 km/sec. Another set of figures is obtained by dividing the length

TABLE II

HEIGHT IN	Gases					
EM	Oxygen	Nitrogen	Hydrogen			
200			3.55			
			3.96			
180			4.41			
70			4.91			
60			5.48			
50		0.0002	6.10			
40		.0008	6.80			
30	1000.0	.0034	7.58			
20	.0004	.0154	8.45			
10	.0022	.0697	9.42			
	.0125	0.315	10.5			
90	.0702	I.43	II.7			
80	0.393	6.45	13.0			
70	2.21	29.2	14.5			
60	12.4	132.	16.2			
50	69.4	597.	18.1			
40	390.	2701.	20.I			
30	2185.	12220.	22.4			
20	12252.	55280.	25.0			

of the path by its estimated duration. The results of these direct determinations show a considerable divergence from the theory; the values ranging from 10 to 160 km/sec. As the duration of visibility is usually a retrospective estimate, made by an observer who is attempting to note at the same time many other features of the phenomenon, it is at least conceivable that such subjective estimates are affected by very large systematic errors. Until such estimates can be checked by instrumental determinations it seems hardly safe to build upon them; a number of writers have, nevertheless, con-

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cluded that a large proportion of the observed meteors come from outside the solar system, and therefore move in hyperbolic orbits. We shall later on present some arguments which seem to make this supposition doubtful; but, in order not to prejudge the matter, a range of speeds from 10 to 120 km/sec. has been assumed in the calculations. The material for testing the theory, however, has been taken entirely from meteors belonging to definite showers; these being members of our own system, their velocities are deducible theoretically.

The height of appearance lies with few exceptions between 160 and 70 km, the general rule being that the faster meteors appear higher, but that for meteors of the same speed the height varies little, if at all, with the magnitude. The very large fireballs and meteorites form a possible exception to this last rule; but as these are usually of the sporadic type not belonging to known radiants, it is difficult to say whether the great heights of appearance sometimes recorded are due to their size or to their having greater velocity.

The height of disappearance is usually below 120 km. Since it is obvious that a meteor which enters the atmosphere horizontally must disappear at very nearly the same height at which it has appeared, this fact only means that at those places at which most observations of meteors have been collected the fastest meteors cannot enter the air from radiants on the horizon. The point is of some interest as indicating the gaps, from the standpoint of the physical theorist, in our meteoric data. There is no definite lower limit to the height of diasppearance. Few of the swifter meteors get below 85 km, the general rule being that the slow meteors appear lower, disappear lower, and describe longer paths than swift meteors of equal brightness.

The apparent temperature of meteors, as indicated by their color, varies from about 7000° K., for fast meteors, to 3000° K., or less, for the slow ones. As the radiation comes, in all probability, from a surrounding mantle of gas, we may not attach any precise quantitative significance to these figures.

We will now take up the details of the theory. In its main features it can be quite simply stated. We consider the meteor as at rest in a stream of gas molecules. At heights between 70 and 160 km the

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free path of an air molecule will be of the same order of magnitude as the assumed dimensions of the particle; there is thus no possible accumulation of air in front of the meteor, but a series of discrete impacts on the solid nucleus. An expression for the rate of input of energy is obtained, which, equated to the rate of radiation according to Stefan's law, gives the temperature of the solid nucleus. The meteor becomes visible when this temperature is high enough to produce copious evaporation.

The details of this theory had been fully worked out before the writer could get access to the paper by Lindemann and Dobson. On reading their paper, he was surprised to find, that, after considering the theory outlined above, they reject it as insufficient. As the whole need for higher atmospheric temperatures and pressures hinges upon this rejection, it would be interesting to have more of the calculations on which it was based. But even granting their postulate of higher pressures and temperatures, the writer is unable to accept the reasoning employed in the development of the theory which they offer as a substitute. Their calculations are vitiated, in his opinion, by a fundamental error. This error lies in their use of the equation of the gas adiabatic to calculate the rate of heating of the meteor. The equations of thermodynamics, including this equation, rest on the fundamental assumption of reversible processes, during which there is at every stage equilibrium of the working substance with its surroundings. To see how far the conditions of the actual problem depart from this requirement, we may consider the picture of an adiabatic compression which is afforded by the kinetic theory. The molecules which strike a moving piston (here the meteor) rebound from it with increased velocity, and by considering the statistics of such molecular impacts, the usual equation for the adiabatic may be deduced. The possibility of this deduction depends, however, upon the assumption that the velocity of the piston is small compared to the temperature velocity of the gas molecules. The use of the adiabatic equation by Lindemann and Dobson is therefore equivalent to the assumption that a velocity of 60 km/sec. is small compared to one of 0.5 km/sec. As a matter of fact, the total energy given to the gas by a definite displacement of the piston, which for small velocities V of the piston is independent of V

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and proportional to T, the temperature of the gas, is for large values of V independent of T, and proportional to V^2 . The first value is that deducible from the adiabatic equation; the second corresponds to the theory of molecular impacts, either directly with the solid nucleus or with the outer boundary of a surrounding gas envelope. To see how large a difference this may make, consider a volume of hydrogen which is compressed to 0.9 of its original value, and which has initially the temperature 273° K. The adiabatic compression gives $T/T_0 = (\rho/\rho_0)^{\gamma-1}$, from which we find $T = 273 \times (10/9)^{0.4} = 285^{\circ}$, a result which holds for all piston velocities which are small compared to 1.8×105 cm/sec. For a piston velocity of 60 km/sec., however, we may consider, in effect, that one-tenth of all the molecules in the gas are suddenly given this velocity. When this translatory energy has been distributed among all the molecules in all the degrees of freedom, the mean-square velocity will be $3/50 \times$ $(6,000,000)^2 = 2.16 \times 10^{12}$. We thus get $T = 273 \times 2.16 \times 10^{12} / (1.8 \times 10^{12})^{12}$ $10^5)^2 = 18,200^\circ$.

This collision of the air molecule with the meteor is a collision with a particular molecule of the meteor. The speed of the air molecule relative to the meteor is so great that the primary transfer of momentum on collision must be to the first thing that is able to start moving. We may go even farther and say that it is probably a question of the collision of atoms, for the energy available will in most cases be far in excess of that required for chemical dissociation. The energy required to dissociate or ionize a substance is $1.57 \times$ 10^{-12} P ergs, where P is the ionization or dissociation potential in volts. Taking for the mass of the hydrogen molecule $m = 3.3 \times 10^{-24}$ gm, we thus get $P = 0.0105 V^2$, where V is in km/sec., as the potential equivalent to the speed V. This gives for V = 16, P = 2.7. This is greater than ordinary lattice potentials, and is of the same order of magnitude as Langmuir's value for the dissociation potential of the hydrogen molecule. For greater speeds or heavier molecules this disruption becomes, a fortiori, a practical certainty. When V = 30 km/sec., we get P = 0.5 volts; this is of the order of magnitude of the atomic ionization potentials. In considering the dynamics of impact, it seems therefore justifiable to regard it as practically inelastic, since this breaking up of the lattice, the molecule, or the atom provides the necessary means for the dissipation of the energy.

In formulating the dynamical problem, we make the usual simplifying assumption of spherical molecules, and consider a molecule of

mass m_2 struck by one of mass m_1 , which is moving relatively to it with velocity V (Fig. 1). Let the line of centers at the moment of impact make an angle θ with the direction of V. Let the component velocities along and perpendicular to the line of centers after impact be u_1, v_1 ; u_2, v_2 . The component of the relative velocity perpendicular





to the line of centers is unaltered by the collision, while that along the line of centers is reduced to zero. We thus have

$$u_1 - u_2 = 0; v_1 - v_2 = V \sin \theta$$
 (1)

We have also two equations of momentum:

$$m_1 u_1 + m_2 u_2 = m_1 V \cos \theta; \quad m_1 v_1 + m_2 v_2 = m_1 V \sin \theta.$$

From these we get

$$u_{1} - u_{2} = \frac{m_{1} + m_{2}}{m_{2}} u_{1} - \frac{m_{1}}{m_{2}} V \cos \theta = V \cos \theta - \frac{m_{1} + m_{2}}{m_{1}} u_{2}$$
$$v_{1} - v_{2} = \frac{m_{1} + m_{2}}{m_{2}} v_{1} - \frac{m_{1}}{m_{2}} V \sin \theta = V \sin \theta - \frac{m_{1} + m_{2}}{m_{2}} v_{2} .$$

Substituting the first set of values in (1), we get u_1 and v_1 . We thus find for the energy of the first molecule after impact

$$\frac{\mathbf{I}}{2}m_{\mathbf{I}}V^{2}\left\{\mathbf{I}-\frac{2m_{\mathbf{I}}m_{2}+m_{2}^{2}}{(m_{\mathbf{I}}+m_{2})^{2}}\cos^{2}\theta\right\}.$$
(2)

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If we substitute the second set of values, we find for the energy of the second molecule

$$\frac{1}{2}m_{1}V^{2}\frac{m_{1}m_{2}}{(m_{1}+m_{2})^{2}}\cos^{2}\theta.$$
 (3)

The sum of these is less than the original energy by the amount dissipated, namely,

$$\frac{1}{2}m_{1}V^{2}\frac{m_{2}}{m_{1}+m_{2}}\cos^{2}\theta.$$
 (4)

We may sum up these results by saying that the first molecule loses on impact a fraction $q_1 \cos^2 \theta$ of its original energy; that it imparts to the second molecule the fraction $q_2 \cos^2 \theta$; and that the fraction q_3 $\cos^2 \theta$ is dissipated, where $q_1 = q_2 + q_3$. We form now the average $\cos^2 \theta$ for all possible collisions. Through the center of m_2 pass a plane perpendicular to the direction of V. At the moment of impact the line of V, drawn through the center of m_1 , meets this plane within a circle whose radius R is the sum of the molecular radii. This circle forms the "target" which m_2 offers to m_1 , and all points within it are equally probable. The collisions for which V meets the target at distances from the center between r and r+dr form a fraction $2rdr/R^2$ of the total. Putting $r = R \sin \theta$, we get $2 \sin \theta \cos \theta d\theta$ as the fraction for which θ lies between θ and $\theta+d\theta$. The mean value of $\cos^2 \theta$ for all collisions is thus

$$\int_{0}^{\frac{\pi}{2}} \cos^2 \theta \cdot 2 \, \cos \theta \, \sin \theta \, \mathrm{d}\theta = 1/2 \; .$$

From a number of analyses of aerolites given by Merrill¹ selected at random from the *Catalogue* of the National Museum, we find a mean atomic weight between 25 and 30. Taking the smaller value as a lower limit, and the atomic weight of iron, 56, as an upper limit, we get the distribution of the energy of collisions shown in Table III.

In the case of hydrogen, the energy of translation which is directly imparted is a very small fraction, so that the question of heating

¹ Smithsonian Inst., U.S.N.M. Bull., No. 94, 1916.

will depend primarily upon what part of the dissipated energy is available for this purpose. Lindemann and Dobson, in their examination of this point, apparently assumed: ($\mathbf{1}$) that all of this dissipated energy immediately appears as radiation; ($\mathbf{2}$) that none of this radiation can be effective in heating the meteor; ($\mathbf{3}$) that even the energy actually imparted to the meteor is ineffective because there is not sufficient time for it to be transmitted to the adjacent molecules—the velocity of elastic waves in the meteor setting an upper limit to this rate of transfer. On this view the effect of the impacts would be to strip off a few molecules at velocities corresponding to an enormously high temperature, instead of using this

TA	RI	E	TI	T
4.4.3	1.1.1	121	**	

	Hydrogen		NITROGEN		
	<i>m</i> ₈ = 25	$m_2 = 56$	m ₂ = 25	ma=56	
Gained by m_1	0.018	0.000	0.25	0.11	
Dissipated	0.48	0.49	0.24	0.33	

energy to evaporate a very much greater number at velocities corresponding to the temperature of evaporation. We whall consider these objections in order.

1. The dissipated energy may be divided into three parts: (a) the energy of dissociation of the atmospheric molecule, (b) the energy of dissociation of the meteoric molecule or of the disruption of its lattice structure, (c) the energy of atomic ionization. Of these, only (c) is responsible for immediate radiation, and we have seen that for speeds less than 30 km/sec. very little of the dissipated energy can come under this head. (a) is clearly not available for heating the meteor but the probably equal amount (b) should go entirely to its credit.

2. The energy radiated proceeds from points at or close to the surface of the meteor. The meteor is thus in the field of its own radiation, and should absorb approximately half of this radiated energy. This figure, which can easily be made to appear too high by arguments having a great deal of plausibility, seems nevertheless to the writer to be the assumption which would naturally be made

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in the absence of any preconception as to the result. It should be remembered, moreover, that, in neglecting the possibility of a second or third impact, the figures for the amount of dissipated energy are certainly too low.

3. The assumption of the velocity of elastic waves as an upper limit for the rate of transfer of energy to adjacent molecules presupposes the existence of this energy as translatory molecular motion at speeds below the limit at which disruption of the lattice occurs. We have available for this transfer, however, not only much higher speeds, but free electrons and radiant energy, giving a speed of transfer comparable to the rapidity of metallic conduction. It is not

	Hydrogen	Nitrogen
$n_2 = 25 \dots \dots$	0.25	0.37
$n_2 = 56$	0.25	0.27

TABLE IV

necessary, for the purposes of the theory, that the distribution of energy should extend to great depths.

We take, therefore, as the energy acquired by the meteor, the total energy of translation *plus* half the dissipated energy. We thus get as the efficiency k of the impact the values shown in Table IV.

The value of k for hydrogen is practically independent of m_2 . For nitrogen we will assume the value 0.37, as the smaller values of m_2 are much more probable.

Assuming the meteor to be a sphere of diameter D, it will be struck in one second by all the molecules in the volume $\pi D^2 V$; that is, by $\pi D^2 V N \rho / \rho_0$, where N is the Avogadro number. From each of these it receives the energy $kmV^2/2$, where m is the mass of the atmospheric molecule. Since $mN = \rho_0$, we thus get for the rate of energy input $\pi k D^2 V^3 \rho / 2$. If we may regard the meteor as a black body radiating at temperature T we have for the rate of loss $4\pi D^2 a T^4$, where a is Stefan's constant (= 5.3 × 10⁻⁵ erg/cm² sec.). Equating this, we get the relation

$$T^4 = k V^3 \rho / 8a. \tag{5}$$

For a mixture of gases $k\rho$ is to be replaced by a summation.
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The meteor will become visible when heated to the temperature of copious evaporation. The writer agrees fully with Lindemann and Dobson that this view seems the only one tenable. If we know this temperature and also the distribution of the atmosphere as a function of the height, we can calculate with the aid of equation (5)the height of appearance corresponding to a given speed. Taking $V = 7 \times 10^6$ cm/sec. and h = 160 km, values which agree well with the observed maximum height of Leonids, and using the values of ρ/ρ_0 tabulated above, we get $T = 3200^\circ$. This is higher than any probable temperature that the substance of the meteor could attain without completely disintegrating. The temperature of rapid evaporation is not definitely known; it is probably in no case above 2500° K., though not less than 2000° K. At these levels, the hydrogen alone is effective; if we assume the value 2500° K., we find for the density of hydrogen 0.4 of the tabulated value; the lower limit 2000° K. requires only 0.16 of the tabulated value. If for brevity we call this temperature the "flashing point" of the meteor, we may sum up these figures thus: On the assumption that the flashing point is between 2000° and 2500° K., and that the height of 160 km corresponds to a speed of 70 km/sec., the theory indicates an amount of hydrogen at this height corresponding to a surface value of between 1 part in 60,000 and 1 part in 25,000 by volume.

The varying composition of meteors will produce corresponding variations in their flashing point. The maximum height would then belong to the fastest and most volatile meteors. If we assume $T=2000^\circ$, V=70, and H=160 to be corresponding values, the amount of hydrogen in the atmosphere is thereby fixed, and equation (5) predicts a definite height of appearance corresponding to any other values of T and V. We thus get the values for h in kilometers as shown in Table V.

If we take Lindemann and Dobson's estimate of $2000^{\circ}-2300^{\circ}$ as the probable limits of T, the range of altitudes will be given by the first two lines. It will be noticed that this range increases with V.

As a rough check on these results we may take Newton's figures for Leonids, 154 km, and for Perseids, 115 km. The maximum velocities are 76 km/sec., and 54 km/sec., respectively. As the figures are averages, the mean velocity would be less than this by an unknown

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amount, but the relative values are in good agreement with the theory. Another more definite figure is represented by the mean height of 3 Geminids, observed by Olivier and Alden.⁴ These gave a mean height of 97 ± 2 km, with a computed speed of 33 km/sec. It does not seem worth while to multiply instances, as the assumed values, from which the table of heights (Table V) was derived, have no great claim to accuracy. The writer would emphasize again the need, for the purposes of a physical theory, of observations restricted to meteors which are known members of the solar system, and to the necessity of using theoretical velocities. This is borne out by the result of the calculations for speeds greater than 70 km/sec. Such meteors, if they exist, should appear at heights much greater than the maximum height of the Leonid meteors. The fact, therefore,

TABLE V	r
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Т						V (kn	n/sec.)					
	10	20	30	40	50	60	70	80	90	100	110	120
2000 [°] 2250 2500	75 72 60	89 86 83	98 94 91	105 101 08	113 107 103	126 114 108	(160) 126 115	197 157 125	232 186 147	259 215 174	285 242 201	309 261 225

that the Leonids appear about as high as any negatives the reality of the high values of some of the direct determinations of speed.

This increase of the height of appearance with the speed seems to be a consequence, not only of this particular theory, but of almost any which could be proposed, besides being well established by observation. The conclusion does not entirely negative the possibility of hyperbolic orbits with radiants of high celestial latitude, but it impugns an important portion of the evidence by which the hyperbolic theory is supported.

When we turn to a consideration of what happens after the meteor becomes visible, the problem is more intricate. Certain rather obvious conclusions may be drawn at once. The light of the visible meteor arises from the excitation of ionization of the escaping vapor by collision with the air molecules. The total amount of radiation emitted will therefore be proportional to the rate of evapo-

¹ C. P. Olivier, Popular Astronomy, 32, 591, 1924.

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ration; while the character of the radiation will depend only upon the levels of excitation or ionization and will be independent of the density of the atmosphere. If a mass E evaporates in unit time, this mass loses by collision the energy $EV^2/2$, a certain fraction of which appears as radiation. It is usually assumed that this fraction must be very nearly unity: this question will be examined later. To confine ourselves for the moment to the qualitative aspects of the question: At heights below the altitude of appearance, the density of the air is increasing. The increasing number of impacts with the naked nucleus would evaporate the meteor almost instantly, but the nucleus is screened from these impacts by the envelope of vapor. When this envelope has reached a sufficient density, few or none of the atmospheric molecules will be able to penetrate directly to the nucleus, but must communicate the necessary energy through the envelope, either by radiation or conduction. It is clear, however, that there will always be evaporation, for as it falls off the rate of supply of energy to the nucleus at once increases because of the relaxation of its defenses. Only in exceptional cases can we imagine it to cease temporarily. A meteor which is moving nearly horizontally might reach, for instance, a temperature of 2000°, at which the more volatile constituents would distil off in sufficient quantity to make it visible. If these should be exhausted before it had reached the level of appreciably greater density, the evaporation would cease for a while, to be renewed lower down. Cases are not unknown in which meteors have thus disappeared and reappeared, but whether the inclination of the path was ascertainable for them, the writer is unable to say. The "spindles," or periodic variations in brightness which are characteristic of photographic meteor trails, might perhaps be similarly explained. The more volatile constituents, distilling off from the surface, would leave a "glaze" of more refractory material. With the reduced rate of evaporation, the temperature would rise, resulting in the evaporation of the refractory film and a sudden increase in the amount of vapor as the underlying fresh material became exposed. It is not clear, however, that the spindles represent variations in the instantaneous brightness of the meteor, or whether they represent dilatations of the luminous train subsequent to the passage of the meteor, so that the explanation just given

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may be fanciful. Similar variations would perhaps be produced by the rotation of a fragment of irregular shape. In the "bursts" and even explosions of the larger fireballs, we seem to be on safer ground, for the known heterogeneity of large meteors makes variation of the rate of evaporation extremely probable.

As we increase V, the ionization or excitation will proceed to deeper and deeper levels, resulting in a general shift of the emitted light toward shorter wave-lengths. The general character of the known relation between speed and apparent temperature is thus easily accounted for. The maximum amount of energy which can be dissipated by a single molecular impact is, according to the collision formulae given earlier, 0.98 times the energy of a hydrogen atom, or 0.66 times the energy of a nitrogen atom; the mean values being half this. By equating these values to $h\nu$, where h is the Planck constant, we obtain a maximum frequency corresponding to a given velocity. Expressed in wave-lengths we thus get in angstrom units:

V =	IO	20	30	40	50	60	70
Hydrogen	23,800	5950	2645	1490	950	660	485
Nitrogen	2550	640	283	160	102	70	52

These figures, even as minima, should probably be doubled, as the energy is divided between the two colliding atoms. They indicate that for hydrogen the speeds below 30 km/sec. would be capable of producing little or no visible radiation. At the levels at which hydrogen predominates, a considerable proportion of the kinetic energy of the vapor would in the course of its collisions become distributed among the hydrogen molecules at speeds too small to excite visible radiation. For the higher velocities, on the other hand, the figures indicate the probability of a quite intense ultra-violet radiation. A portion of this, consisting of wave-lengths less than 1000 A, would be reabsorbed by the hydrogen, and would reappear in part as visible light. Meteoric masses have usually been calculated by taking the total radiated energy corresponding to a given stellar magnitude, assuming the sun as a comparison star, and equating this to the kinetic energy of the meteor. This virtually assumes an efficiency of radiation equal to that of the sun; if the sun is Q times

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as efficient, these masses should be multiplied by Q. The value of Q cannot be accurately determined, but an approximate figure may be found by comparing the efficiency of the best artificial sources with that of the sun. The flaming arcs form a fair experimental analogue of the processes of meteoric luminescence, but the efficiency of the best of them (2.5 candles/watt) is only one-third that of the sun (7 candles/watt). The efficiency of the meteor's luminescence may very well be less than this, as, owing to the high velocities of collision, its spectrum would approach the spark type, at least in the case of collisions with nitrogen. It would seem safe to assume, therefore, values of Q at least as great as 5 and perhaps even 25.

Since the sun, a star of magnitude -26.7, sends to the earth about 1.4 \times 10⁶ erg/cm² sec., we get for a star of magnitude M about $2.8 \times 10^{-5} \times (2.5)^{-M}$ erg/cm² sec. This gives, for the total radiation of a meteor of this magnitude at r km distance, $3.5 \times 10^6 \times (2.5)^{-M} r^2$ erg/sec., which we shall call Br^2 . The total kinetic energy required to produce this radiation is OBr^2 , so that we have $OBr^2 = 2\pi\rho' V^2 D^2 dD$ /dt (ρ' = density of meteor). It is generally agreed that the smaller meteors show little, if any, change in apparent brightness during their entire course; this would require the constancy of $(D^2 dD/dt)/$ r^2 , since V is known to change very little. D is, however, diminishing; r probably also diminishes somewhat for most observed meteors, but not rapidly, so that dD/dt should increase. It is difficult to conceive any process by which the constancy of $D^2 dD/dt$ is assured under all circumstances. The rate of evaporation of unit area is measured by dD/dt. It is reasonable to suppose that it increases with the density of the air, but it is difficult to see why it should depend upon anything else except V. We might imagine, in a particular case, that the diminution of D and the increase of dD/dtwere so adjusted as to give a roughly constant rate of mass diminution, but then the assumption of a greater initial diameter would give a rapid increase in brightness, while the assumption of a more inclined path would result in a diminution. It must be admitted that the constancy of magnitude cannot be accurately affirmed. The estimates of brightness by different observers frequently vary by a whole magnitude. If we exclude from consideration the faintest meteors whose paths are too short to afford any accurate judgment,

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and the larger meteors which undoubtedly do show variations of brightness, the question hinges on the facts concerning magnitudes between 4 and 0, and it may be that the constancy is best for meteors of the second or third magnitudes, and even then not so good as has been supposed. If this constancy should be confirmed, it would then be necessary to understand why, as the mass of the nucleus diminishes, the surface rate of evaporation increases. One or two other points which bear on this question may be mentioned, which will serve mainly to show the extreme complexity of the problem. The rate of evaporation is a very sensitive function of the temperature. For the larger meteors, the conduction of heat to the colder interior of the nucleus tends to lower this: the smaller meteors tend to get heated through, thus diminishing this two-sided loss of the heated surface laver. If all the vapor emitted were effective in screening the meteor, this would perhaps be unable to increase evaporation, but only that which comes off from the advancing face can protect the nucleus, so that for the smaller meteors there would be a larger proportional loss of mass.

Let us consider now some details of the formation of the luminous envelope. The quantity QBr^2 , which we equated above to the kinetic energy of the vapor lost per second, can be also considered as the rate of input of energy into this vapor by the collision of the air molecules. Each of these has relative to the meteor the speed V; to contribute the necessary energy a certain number is required. Since the meteor traverses the distance V, we may consider these molecules as contained in a cylinder of diameter D' and length V. D' may be called the effective diameter of the envelope; it is the least diameter of the track within which the luminous molecules must lie. We thus get

$(\pi/8)\rho D'^2 V^3 = QBr^2$.

The difficulty in calculating D' lies in the uncertainty of Q. Even for small values of Q, however, the equation gives values of D' much larger than the diameter of the nucleus as calculated in the usual way. This would indicate that the actual envelope takes the form of a short spreading tail, like the tail of a comet, since in the time required for the lateral migration of the vapor it would be left

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behind by the meteor. Again we may compare the values of D' with the diameter of a circular disk, which, radiating like a black body of the same color temperature, would appear of equal stellar magnitude. Even assuming that the envelope is spherical, this gives for the solid disk diameters from a third to a fourth of the corresponding values of D'. This is as it should be, for at such low densities we should not expect the envelope to be perfectly opaque.

That the meteor does not sensibly slow up throughout its course is an immediate consequence of the foregoing. If L is the length of the meteor's path, its total energy is roughly equal to that of the air molecules in the volume $(\pi/4)D'^{2}L$, considered as moving with the speed V. Only those in the volume $(\pi/4)D^{2}L$ can possibly be effective in stopping the nucleus, for the other molecules collide with the vapor to the side and rear. The direct loss of energy is certainly not greater than the fraction D^{2}/D'^{2} of the whole. It must, in fact, be considerably smaller, for the collisions in the line of the nucleus are not direct impacts on the nucleus, but with its vapor, which is being swept aside. The effect of evaporation is to provide, as it were, a slippery sheath within which the nucleus is able to progress with very slight retardation.

Let us return now to a consideration of the process by which, after the formation of the envelope, the evaporation of the meteor is maintained. The writer's attempts at a mathematical formulation of this problem have so far been unsuccessful; a clear statement of the physical picture will be all that can be attained. The outer part of the envelope, which receives the impacts of the atmospheric molecules, will consist of a mixture of air and vapor. Referred to the meteor, the mean velocity of these molecules is high, but it is only in a loose sense that we can identify this mean velocity with a temperature, since the law of distribution departs widely from Maxwell's law. There is nevertheless a transition, as we go from these outer parts of the envelope in to the nucleus, from a region of high mean molecular energy to a region with a very much lower value. There will thus be a transfer of energy by molecular collisions by a process identical, from the point of view of kinetic theory, with that of heat conduction. It differs from heat conduction only in that the ordinary formulae will not apply, for the energy of the molecular

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motion cannot be called heat. As the same mechanism of transfer is invoked, however, we should expect the rate of transfer to be independent of the density of the vapor, and to be proportional to the difference in mean molecular energy at the boundary and at the surface of the nucleus, and to vary inversely as the thickness of the nucleus. This quasi-conductivity of the envelope will be much increased by the free electrons present, which by virtue of their greater free path and high speed will carry most of the energy. In addition, we have to include the energy received by the nucleus from the radiation of the envelope. This is perhaps not large; for the radiation comes for the most part from that portion of the envelope for which the density of the vapor is small, while close to the surface of the nucleus the denser and colder layers of vapor would probably absorb most of the ingoing radiation.

To complete this picture we must examine the process at heights below 60 km, where the free path is small enough to permit the formation of an air-cap. The main differences thus introduced seem to be two. First, the screening of the nucleus is not dependent, as before, on the rate of evaporation; and second, the luminescence is more a matter of the ionization of air by air, for it is the piled-up air that now receives the impacts of the stream of fresh molecules. Whether this would result in any essential reduction of the rate of evaporation is difficult to say. At this point, also, the effects of a true viscous drag between the gas-cap and the surrounding air would begin to be felt, and would increase as the height diminished, so that the deceleration would be more marked. But in calculating the rate of input of energy the same principles should apply as before; the idea of adiabatic compression being no more applicable here than at the greater heights, we have still a rate of input which is proportional to V^3 , and which is practically independent of the temperature of the atmosphere.

Rouss Physical Laboratory University of Virginia November 15, 1925

ON SELECTIVE RADIATION PRESSURE AND THE ACCELERATED MOTION OF Ca⁺ VAPOR IN ERUPTIVE PROMINENCES

By RAMANI KANTO SUR

ABSTRACT

An attempt has been made to explain the motion of eruptive prominences.

The great heights reached by certain elements in the solar chromosphere cannot be explained by the theory of a pressure gradient due to solar gravity or by any electrical theory. On the assumption that the ionized calcium vapor in the high-level chromosphere is supported against gravity by radiation pressure, Milne has given, by applying Einstein's theory of radiation, a calculation of the average life of an excited Ca+atom in the 1s state which agrees well with the experimental results of others. These ideas have been extended to show that an *increase of temperature* of any portion of the photosphere above the surrounding parts will destroy the equilibrium of the gaseous mass above it. The excess of the radiation pressure over the solar gravity produces an acceleration, the magnitude of which depends upon the temperature, the shape, and the size of the bright patch developed and the distance of the gaseous mass under consideration. The molive power for the motion of eruptive prominences is thus furnished by selective radiation-pressure.

According to Einstein's theory of radiation, the moving mass of gas will experience a force of resistance proportional to velocity which will reduce the motion to one of constant limiting velocity. The impulsive changes of velocity are due to sudden changes of temperature at the base.

INTRODUCTION

Very recently Professor Milne¹ has given from astrophysical data a very simple and interesting calculation of the average life of an excited Ca^+ atom. It is well known that the H and K lines of Ca^+ occur in the highest levels of the chromosphere, much higher than the level reached by the much lighter hydrogen and helium. If atoms in the atmosphere of the sun were subject to gravitational forces alone, then, remembering that the value of gravity on the surface of the sun is 27.6 times that on the earth, it is easily seen that the density ρ of an element of atomic weight A at a height of z kilometers is roughly given, on the isothermal theory, by $\rho = \rho_0 \cdot 10^{-\frac{Az}{400}}$, ρ_0 being the density on the surface. Thus at a height of 8000 kilometers the density of hydrogen would be 10^{-20} times its value on the surface of the sun, and that of calcium would be 10^{-800} times its density on the surface. Supposing ρ_0 to correspond to 10^{19} in the case of hydrogen

¹ Monthly Notices of the Royal Astronomical Society, 84, 354, 1924.

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as well as of calcium (this is now known to be an overestimate), we shall get at a height of 8000 kilometers scarcely one atom of hydrogen per cubic centimeter, and not a single atom of calcium in the whole space at this height over the disk. But observations of the flash spectrum show that the H and K lines of Ca^+ reach a level of 14,000 kilometers, while the Balmer lines of hydrogen are strong even at a height of 8000 kilometers. It is, therefore, apparent that, in the atmosphere of the sun, gravity is reduced by certain repulsive forces to a very small fraction of its commonly accepted value, and that these repulsive forces act selectively on different kinds of atoms.

A few years ago, Saha suggested that these repulsive forces were due to radiation pressure acting in a selective manner. These ideas have now been put by Milne in a quantitative form.

I. THE POSSIBLE EFFECT OF ELECTRICAL FORCES

The origin of this repulsion may also be looked for in forces of electrical origin. It is, therefore, first necessary to examine the possible influence of these. The atmosphere of the sun, as well as of other stars, is highly ionized on account of high temperatures, and electrons may be constantly escaping to surrounding space. Milne calculates that the sun may in this way acquire a maximum positive potential of 30 volts. But whatever may be the magnitude of these electrical forces, they do not appear in any way to be effective in giving rise to the "force of levity." For electrical forces can act only on ionized atoms, and not on neutral atoms. Now it is not only the ionized atom of Ca^+ , but there are some neutral atoms which are also subject to this "force of levity." Leaving the doubtful case of hydrogen, we can cite those shown in Table I. These are resonance lines of neutral elements, and their great heights are unaccountable on any electrical theory.

On the other hand, it is quite possible that owing to local concentration of electrons or positive charges, local fields of great intensity may sometimes be produced. These local fluctuations may cause modification in the act of absorption and emission, probably resulting in a broadening of the lines, but they cannot influence the phenomena treated here in any way.

RADIATION PRESSURE IN PROMINENCES

II. SELECTIVE RADIATION PRESSURE

The way in which selective radiation pressure acts on an atom has been treated by Einstein^t and by Saha.² When a pulse of light of frequency ν falls upon an atom and is absorbed, the atom suffers

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-		-	4.4	4.4	-	

LINES OF NEUTRAL ATOMS REACHING GREAT HEIGHTS IN THE CHROMOSPHERE

	-	SERIES	IN	HEIGHT OF		
LINE	ELEMENT	DESIGNATION		Chromosphere	SPHERE IN KILOMETERS	
3824	Fe*	$d_1 - d'_2$	6	8	1000	
3856		d2-d'	9	IO	6000	
3860		$d_{\mathbf{i}} - d'_{\mathbf{i}}$	20	20	6000	
3878		d3-d'	II	15	1200	
3886		$d_2-d'_2$	15	15	1600	
3895		$d_4 - d'_5$	7	4	1200	
3899.7		d3-d3	8	3	1000	
3906		d4-d'	10	2	750	
3920		ds-d'	IO	6	1000	
3923		$d_2 - d'_1$	12	8	1200	
3928		$d_4 - d'_3$	8	IO	1000	
3930		$d_3 - d_2'$	8	8	1000	
4226.9	Ca	1S-2P	20	25	5000	
5890 (Da)	Na	15-2p1	30	IO	1000	
5896 (D1)		15-2p2	20	10	1000	
3944	Al	2p2-15	15	15	2000	
3961		2p1-15	20	20	1500	
3829	Mg	2p3-3d	10	20	6000	
3832		2p2-3d	15	30	6000	
3838		2p1-3d	25	40	7000	
5172		2 p2-15	20	20	1000	
5183		2p1-15	30	25	1200	

* Resonance lines of iron according to Laporte, see Zeitschrift für Physik, 23; see also Gieseler and Grotrian, *ibid.*, 22, 249. The heights are taken from Mitchell's list of the chromospheric spectrum. Lines λ 38,56 and λ 3860 are strong in the flash spectrum, though they are not enhanced lines. This is due to Fe possessing a very large ionization potential, viz., 8.15 volts according to Grotrian. But even this seems to be an underestimate.

a forward kick of momentum $h\nu/c$ in the direction of propagation of the pulse. When it emits the light of frequency ν , it suffers a backward kick of momentum $h\nu/c$. According to the above authors, both these processes are directed ones; for if a pulse were emitted, as

¹ Physikalische Zeitschrift, 18, 121, 1917.

² Astrophysical Journal, 50, 220, 1919; Journal of Science, University of California, February 1920.

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Sommerfeld¹ holds, in a spherical wave, the momentum of the backward kick would vanish, and the laws of black body radiation cannot be deduced from statistical mechanics, as has been done by Einstein. Now for an atom in the atmosphere of the sun, the pulses which are absorbed all come from the sun; hence their action is cumulative. On the other hand, if the atoms are isotropic, the pulses are emitted in random directions, so that their aggregate effect is nil. The net result amounts to a forward acceleration of value $\frac{1}{2}(h\nu/mc)\tau$, where τ is the number of times a pulse is absorbed per second by a particular atom of mass m.

Milne takes, as a specific example, a Ca^+ atom in the 1σ state and subject to H and K radiations from the sun. The normal state of the Ca^+ atom is denoted by 1σ , and the excited states corresponding to H and K absorption are denoted by $2p_2$ and $2p_1$ respectively. Let N_1 and N_2 denote the number of Ca^+ atoms in the 1σ and any of the excited states, respectively, per unit volume of the chromospheric space. Following Einstein we have:

$$dN_2 = -dN_1 = -N_2 A_{2 \rightarrow i} dt + \rho_{\nu} [N_1 B_{1 \rightarrow 2} - N_2 B_{2 \rightarrow i}] dt . \tag{I}$$

 $dN_2 =$ total increase in the number of N_2 atoms, which is made up of: (1) $-N_2A_{2\rightarrow 1}dt$ atoms which pass from 1π to 1σ state under spontaneous emission. $A_{2\rightarrow 1}$ is the probability that an atom in the 1π state will pass by spontaneous emission into 1σ state. (2) $\rho_{\nu}N_{1}B_{1\rightarrow 2}dt$ atoms which pass from 1π to 1σ state under absorption of ν radiation of density ρ_{ν} . $B_{1\rightarrow 2}$ is the probability factor of absorption under the action of light of unit density. (3) $-\rho_{\nu}B_{2\rightarrow 1}N_{2}dt$ atoms which pass from 1π to 1σ state by stimulated emission (Einstein's negative Einstrahlung) under the influence of radiation of density ρ_{ν} . $B_{2\rightarrow 1}$ is the probability factor for stimulated emission under the action of light of unit density.

If τ_{I} denote the average life of a Ca^{+} atom in the $I\sigma$ state and τ_{2} the average life in the $I\pi$ state, then equation (I) may be put in the form

$$dN_{a} = -dN_{I} = -\frac{N_{a}}{\tau_{a}}dt + \frac{N_{I}}{\tau_{I}}dt . \qquad (I')$$

* Atombau und Spektralanalyse, 2d edition, chapter on the Auswahlprinzip.

By comparison

7

$$\tau_1 = \frac{\mathbf{I}}{B_{1 \to 2} \rho_{\nu}}; \qquad \tau_2 = \frac{\mathbf{I}}{A_{2 \to 1} + B_{2 \to 1} \rho_{\nu}}. \tag{2}$$

Hence

$$\frac{\tau_1}{\tau_a} = \frac{A_{2 \to 1} + B_{2 \to 1} \rho_{\nu}}{B_{1 \to 2} \rho_{\nu}} \,. \tag{3}$$

Now $p_1 B_{1 \rightarrow 2} = p_2 B_{2 \rightarrow 1}$, p_1 and p_2 being the weights of the two states under consideration; from Einstein's theory, we have

$$\frac{A_{2 \rightarrow 1}}{B_{2 \rightarrow 1}} = \frac{8\pi h\nu^3}{c^3} .$$

Thus

$$\frac{\tau_1}{\tau_2} = \frac{p_1}{p_2} \left[\frac{8\pi h\nu^3}{c^3 \rho_\nu} + \mathbf{I} \right] \,. \tag{3'}$$

In the high-level chromosphere, the atoms are exposed to radiation from the sun on one side only. Then

$$\tau_{I} = \frac{I}{\frac{1}{2}B_{I \rightarrow 2}\rho_{\nu}}; \qquad \tau_{2} = \frac{I}{A_{2 \rightarrow I} + \frac{1}{2}B_{2}\rho_{\nu}}$$
(4)

$$\frac{\tau_1}{\tau_2} = \frac{p_1}{p_2} \left[2 \frac{8\pi h \nu^3}{c^3 \rho_p} + \mathbf{I} \right].$$
 (3")

 $\rho_{\nu} = r \cdot \frac{8\pi h\nu^3}{c^3} \cdot \frac{\mathbf{I}}{\frac{h\nu}{e^{kT} - \mathbf{I}}}, \qquad (5)$

r is the ratio of the intensity in the center of the line to that in the adjacent continuous spectrum, and the photospheric radiation is assumed to be that of a black body at temperature T. From (3'') and (5),

$$\frac{\tau_1}{\tau_2} = \frac{p_1}{p_2} \left[\frac{\frac{kr}{e^{kT} - 1}}{\frac{1}{2}r} + 1 \right]. \tag{6}$$

For equilibrium the impulse of gravity in time $\tau_1 + \tau_2$ must be equal to the impulse of radiation pressure. In time $\tau_1 + \tau_2$ only one pulse of light is absorbed; but the momentum gained in the vertical direction by the absorption of a pulse in the direction θ is $(h\nu/c) \cos \theta$,

Also

or

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the average of which over the surface of the sun facing the atom is $\frac{1}{2}h\nu/c$. Thus we have

$$mg(\tau_1 + \tau_3) = \frac{1}{2} \frac{h\nu}{c};$$
 (7)

m denotes the mass of the absorbing atom. Since τ_2 is small in comparison to τ_1 , we may write equation (7) as

$$mg\tau_1 = \frac{1}{2}\frac{h\nu}{c} . \tag{7'}$$

Combining the equations (6) and (7'), we obtain the value of τ_2 .

Generally we take $p_x = p_2$. Milne, taking a mean value of ν corresponding to λ 3950 and $T = 6000^{\circ}$, obtains $\tau_x = 4.6 \times 10^{-5}$ sec. and $\tau_2 = r \times 5.4 \times 10^{-8}$ sec.; r = 0.11. The value of τ_2 is quite in accordance with the experimental results of Wien^r on the average life of activated atoms.

III

The object of the present paper is to show that these ideas are quite sufficient to explain another connected phenomenon, viz., that of the accelerated motion of Ca^+ vapor in eruptive prominences. The idea was first given by Saha in a qualitative way, but Milne's method enables us to give a quantitative form to it.

The eruptive prominences are huge masses of high-level chromospheric elements which spring into existence with the rapidity of an explosion and move radially outward with an accelerated velocity and dissipate into space after a brief career. An eruptive prominence observed by Evershed at Kodaikanal was found to shoot at $8^{h_{21}m}$, with radial velocity of 79 km/sec. The velocity gradually increased and amounted to 292 km/sec. at $8^{h_{55}m}$; from this we conclude that the prominence moved with a mean acceleration of $\frac{1}{9}$ km/sec.² or 0.4 that due to solar gravity.

It thus appears that gravity was not only neutralized but also reversed by the sudden development of some repulsive force. Evershed² gives the following description of such prominences:

There is some evidence that eruptive prominences consist in their earlier stages of unusually dense low lying gas giving strong absorption in Ha and

¹ Annalen der Physik., 69, 325, 1922.

² Bulletin of the Kodaikanal Observatory, 3, 209, 1917.

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the calcium H and K lines. The mass of gas may persist for several days apparently unchanged, and then become unstable, coming under the influence of a force which tears it into shreds and sends the fragments flying into space with accelerating speed.

Now the sudden development of the repulsive forces on the solar disk cannot be due to the arrival of hot masses of vapor from the interior, as was pointed out by Pringsheim¹ and by Strutt;² nor will the hypothesis of an electrical force explain it, because the repulsive forces act not only upon ionized particles but also upon neutral atoms (Ha line). On the other hand, Evershed's observations clearly point out that the velocity is acquired outside the photosphere and is of the nature of accelerated speed. It has been found that all eruptive prominences have filaments and faculae as their bases. These filaments and faculae are generally brighter than the rest of the surface and are, therefore, regions of higher temperatures. This view was put forward by Saha, who maintained that the spectra of faculae would be more like F-type than like solar-type stars. The hypothesis has been confirmed by the observations of spectra of faculae by St. John.³ The explanation, therefore, is quite evident. We have seen that, according to the views put forward before, the gravitational force on a high-level Ca^+ atom is neutralized by the selective radiation pressure arising from the continued absorption of pulses of H and K radiation by the Ca^+ atom. If, however, more pulses pour in for absorption by the Ca^+ atom, the selective radiation pressure will be increased so much that the gravitation is not only neutralized, but also reversed. It is, moreover, possible to give a calculation.

We shall suppose that the facula is in the form of a circular patch at temperature $T_{\mathbf{x}}$ and subtending an angle 2θ at the Ca^+ atom in question; i.e., the solid angle subtended is $2\pi(\mathbf{1} - \cos \theta)$. If $\tau'_{\mathbf{x}}$ be the average life in the lower quantum state,

$$\tau'_{\mathbf{z}} = \frac{\mathbf{I}}{\left[\sin^2\frac{\theta}{2}\rho'_{\mathbf{y}} + \left(\frac{1}{2} - \sin^2\frac{\theta}{2}\right)\rho_{\mathbf{y}}\right]B_{\mathbf{z} \to \mathbf{z}}}$$

¹ Physik der Sonne, p. 225.

² Monthly Notices, Royal Astronomical Society, 77, 59, 1916.

³ Physical Review, 19, 390, 1922.

(8)

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and

$$\frac{\tau_{1}'}{\tau_{1}} = \frac{\frac{1}{2}e^{-\frac{h\nu}{kT}}}{\sin^{2}\frac{\theta}{2}e^{-\frac{h\nu}{kT_{1}}} + \left(\frac{1}{2} - \sin^{2}\frac{\theta}{2}\right)e^{-\frac{h\nu}{kT}}}$$
(9)

$$=\frac{1}{1+2\sin^2\frac{\theta}{2}\left[e^{-\frac{h\nu}{k}\left(\frac{1}{T}-\frac{1}{T_i}\right)-1}\right]}$$

$$=\frac{\mathbf{I}}{H-\cos\theta\cdot(H-\mathbf{I})}\tag{9'}$$

where $H = e^{\frac{h\nu}{k} \left(\frac{i}{T} - \frac{i}{T_s}\right)}$.

-

Now the average value of $\cos \theta$ over the solid angle $2\pi(1-\cos \theta) = \cos^2(\theta/2)$ and over the rest of the hemisphere $=\frac{1}{2}\cos \theta$. Further, the amount of radiation or the number of pulses in a definite solid angle may be taken proportional to the solid angle and to the density of radiation, i.e., to $\Omega \rho_r$, Ω being the solid angle. Thus the average momentum acquired in the vertical direction by absorption of a pulse of light is

$$= \frac{2\pi(\mathbf{I} - \cos\theta) \cdot \rho_{\nu}' \cdot \cos^2\frac{\theta}{2} + 2\pi\cos\theta \cdot \rho_{\nu} \cdot \frac{1}{2}\cos\theta}{2\pi(\mathbf{I} - \cos\theta)\rho_{\nu}' + 2\pi\cos\theta \cdot \rho_{\nu}} \cdot \frac{h\nu}{c} \quad (10)$$
$$= \frac{h\nu}{e^{\frac{h\nu}{kT}}} \cdot (\mathbf{I} - \cos\theta)\cos^2\frac{\theta}{2} + \frac{h\nu}{e^{\frac{h\nu}{kT}}} \cdot \frac{1}{2}\cos^2\theta}{h\nu}$$

$$= \frac{1}{2} \frac{H - (H - \mathbf{I}) \cos^2 \theta}{H - (H - \mathbf{I}) \cos \theta} \cdot \frac{h\nu}{c} .$$
 (10')

C

The net average momentum gained in the forward direction by the absorption of a pulse

$$= \frac{1}{2} \frac{H - (H - \mathbf{I}) \cos^2 \theta}{H - (H - \mathbf{I}) \cos \theta} \cdot \frac{h\nu}{c} - mg(\tau_1' + \tau_2) .$$

 $e^{\overline{k}T}(1-\cos\theta)+e^{\overline{k}T}\cos\theta$

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Neglecting τ_2 , this may be taken equal to

$$\frac{1}{2} \frac{H - (H - \mathbf{i}) \cos^2 \theta}{H - (H - \mathbf{i}) \cos \theta} \cdot \frac{h\nu}{c} - mg\tau_{\mathbf{i}}'$$

$$= \frac{1}{2} \frac{H - (H - \mathbf{i}) \cos^2 \theta}{H - (H - \mathbf{i}) \cos \theta} \cdot \frac{h\nu}{c} - mg \cdot \frac{\mathbf{i}}{H - (H - \mathbf{i}) \cos \theta} \cdot \tau_{\mathbf{i}}$$

$$= \frac{mg\tau_{\mathbf{i}}}{H - (H - \mathbf{i}) \cos \theta} \{(H - \mathbf{i}) \sin^2 \theta\},$$

$$= mg\tau_{\mathbf{i}}' \{(H - \mathbf{i}) \sin^2 \theta\}.$$
(11)

This expression shows that the atom would be repelled with a force equal to $(H-1) \sin^2 \theta$ times the gravity.

We now give two numerical examples.

1. If $T = 6000^{\circ}$ K.; $T_x = 7500^{\circ}$ K. (temperature of the faculae), $\theta = 30^{\circ}$; and the mean value of ν corresponds to λ 3950,

$$(H-1)\sin^2\theta=0.59$$

or the prominence would be repelled with an acceleration of 0.59 times the solar gravity. The value of the repulsive acceleration will, however, decrease, as the prominence shoots outward, owing to decrease in the value of θ .

2. If $T = 6000^{\circ}$ K., $T_1 = 7500^{\circ}$ K., and $\theta = 15^{\circ}$,

$$(H-1)\sin^2\theta=0.16$$

or the prominence would shoot outward with an acceleration = 0.16 times the solar gravity.

It is to be noted that the value of θ will depend upon the height of the point in the prominence under consideration. If *h* be the height, then the radius of the patch = *h* tan θ . For different heights *h*, the repulsive acceleration is

$$(H-1)\frac{r^2}{r^2+h^2}$$
 × solar gravity.

When h is very large compared to r, the repulsive acceleration would vary inversely as h^2 . Probably it is dissipated into obscurity before this stage is reached.

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For complicated forms of the bright patches, the calculation would of course be rather difficult.

The above considerations will probably suffice to show that the motion of eruptive prominences may be explained from the standpoint of selective radiation pressure.

In conclusion the writer wishes to record his thanks to Professor M. N. Saha for suggesting the problem and for his interest in the work.

Postscript (added during correction of proofs).—After this paper had come through the press, the editor of the Astrophysical Journal very kindly invited my attention to a thesis by Edison Pettit on the "Forms and Motions of the Solar Prominences," (Publications of the Yerkes Observatory, Vol. III, Part IV, 1925). Mr. Pettit undertook a very thorough analysis of the motion of prominences photographed with the Rumford Spectroheliograph of the Yerkes Observatory. He concludes that the motion of the eruptive prominences is in most cases not accelerated, but uniform, the increment in velocity taking place in impulses. Mr. Pettit examines the various theories, (I) electrical, and (2) theories based on radiation-pressure, and inclines to favor a sort of electrical theory.

It is not possible to deal in detail with the conclusions of Mr. Pettit in this short postscript. But any electrical theory must be discarded, because these high velocities are observed not only in the case of Ca^+ lines H and K, but also in the case of hydrogen lines Ha, $H\beta$, and $H\gamma$. The emitters of these lines are neutral atoms of hydrogen; hence no electrical force can act on them. Similarly it is impossible to explain the high level acquired by lines of Fe and other neutral elements on any electrical theory.

The second point is whether the theory of selective radiationpressure affords a satisfactory explanation of the phenomena. We have seen above that, if any portion of the photosphere gets heated in excess of the surrounding parts, a bright patch appears, and the equilibrium of the gaseous mass above it will be destroyed. The atoms emitting resonance lines will be subjected to an acceleration depending upon the geometrical shape of the bright patch and the difference of temperature. There is no reason why the motion should be exactly normal, for if two or more bright patches act simultane-

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ously, the motion of the gaseous mass may be inclined at any angle to the normal.

A difficulty arises owing to the fact that according to Pettit's analysis the motion seems to be uniform. According to well-known laws of dynamics, a particle subjected to a constant acceleration can acquire a constant velocity if it moves in a viscous medium. The equation of motion is

$$m\frac{dv}{dt} + Rv = E$$

The terminal velocity acquired is given by E/R.

Now according to Einstein's theory of radiation, a resonator moving in a field of radiation experiences a frictional force opposite to its motion and proportional to its velocity. The coefficient

$$R = \frac{8\pi h^2 \nu^4}{3c^5} \frac{x}{e^x - \mathbf{i}} \frac{B_{\mathbf{i} \to 2}}{\mathbf{i} + \frac{p_m}{p_n} e^{-h\nu/kT}}$$
$$x = \frac{h\nu}{kT}$$

With certain assumptions regarding the values of $B_{1\rightarrow 2}$ and p_m , p_n , and supposing that the force E, whose magnitude is $mg(H-1)\sin^2\theta$, does not vary within wide limits for distances considered, it appears that this frictional force is quite sufficient to reduce the motion to one of constant limiting velocity.

Mr. Pettit has found that the velocity is subject to impulsive increments. According to the present theory, any fluctuation of temperature at the base will produce a corresponding change in the value of E; and the velocity will be proportionately altered. Thus impulsive changes in velocity must be ascribed to sudden changes in temperature at the base.

I hope to deal in detail with the matter in a future communication.

DEPARTMENT OF PHYSICS Allahabad University Allahabad, India

PHOTOGRAPHIC STUDIES OF NEBULAE FOURTH PAPER¹

By JOHN CHARLES DUNCAN

ABSTRACT

Studies of the form and structure of diffuse nebulae, both luminous and obscure, from photographs made with the roo-inch and 60-inch reflectors. Halftone reproductions are given of photographs with the Hooker telescope of the dark nebula Barnard 86 and of uncatalogued nebulosities near α and γ Cygni. The nebulosity between α Cygni and the America nebula is found to possess a number of interesting features. Two nebulous stars are found in the neighborhood of γ Cygni. The spectra of one of these and of H.D. 195255 situated at the edge of a dark marking in the nebula east of γ Cygni are of late B type. The spectrum of H.D. 190178, situated on the line dividing a bright nebula from a dark one, is of type G5, and its spectroscopic parallax, determined by Mr. Adams, is 0.015.

The study of the forms of the nebulae has been continued, mainly with the 100-inch Hooker telescope, at intervals since the publication of the last paper² of this series. Special attention has been given to the diffuse nebulae, both luminous and obscure, in the constellations of Sagittarius and Cygnus.

The Dark Nebula Barnard 86 Sagittarii

$a = 17^{h} 58^{m} 2, \quad \delta = -27^{\circ} 52' (1925)$

- Negative Δ 180, Hooker telescope, 1921, June 6. Seed 30 plate, exposure 2^h. Seeing poor, images large and elongated by atmospheric dispersion. Illustrated in Hale's *The Depths of the Universe*, page 47
 - Δ 249, Hooker telescope, 1925, July 19. Eastman Speedway plate, exposure 2^h30^m. Seeing fair, images elongated. Illustrated in Plate VI

Discovered in 1883 with a 5-inch refractor by Barnard, who describes it as "like a drop of ink on the bright sky,"³ this object is probably the most striking in its visual appearance of all the dark nebulae. Especially so is the pocket at the south-preceding end, the edge of which is very sharp and which contains only two tiny stars, both of them too faint to be detected visually. The eastern border of the dark body is much less clearly defined than the western. About 10'

- ¹ Contributions from Mount Wilson Observatory, No. 303
- ² Mt. Wilson Contr., No. 256; Astrophysical Journal, 57, 137, 1923.

3 Publications of the Lick Observatory, 11, Plate 50, 1913.



E

PLATE VI

THE DARK NEBULA BARNARD 86

Photographed with the roo-inch Hooker telescope, 1925, July 19 Exposure, $2^h 30^m$ Scale: 1 mm = 8% 6 (1.84 times that of original negative)

Negative A 240

M



PLATE VII

Ν



Negative $\Delta 252$

NEBULOSITY PRECEDING THE AMERICA NEBULA IN CYGNUS Photographed with the 100-inch Hooker telescope, 1925, July 20 Exposure, $4^{h}45^{m}$ Scale: 1 mm = 14."2 (1.11 times that of original negative)



PHOTOGRAPHIC STUDIES OF NEBULAE

east of the center of the dark nebula is the bright open cluster N.G.C. 6520, and south of this, as shown in the illustration, is a dark marking that is less pronounced than the main nebula, and is not noticeable visually. The brightest star in the immediate neighborhood is H.D. 164562, visual magnitude 6.68, spectrum Ko, situated on the north-preceding border of Barnard 86. It is of a deep orange-red color that is very striking when seen in the great reflector. In the bright star-cluster also are a number of orange-colored stars. The background against which the dark nebula is seen seems to be composed of faint stars, and there is no evidence on the negatives of any bright nebulosity.

At the latitude of Mount Wilson $(+34^{\circ}13')$, objects having so great a south declination as this are always so near the horizon that the star-images, drawn out into short spectra by atmospheric dispersion, appear elongated on all plates made with the Hooker telescope.

A photograph made by H. D. Curtis with the Crossley reflector is reproduced in *Publications of the Lick Observatory*, **13**, Plate VII, **1918**.

Portion of the Faint Nebulosity Preceding the North America Nebula (N.G.C. 7000 Cygni)

 $a = 20^{h}49^{m}7, \delta = +44^{\circ}7' (1925)$

Negative Δ 248, Hooker telescope, 1925, July 18. Aperture 84 inches. Eastman 40 plate, exposure 3^h30^m

 Δ 252, Hooker telescope, 1925, July 20. Eastman Speedway plate, exposure $4^{\rm h}45^{\rm m}$. Illustrated in Plate VII

The existence of faint nebulosity west of the America nebula is shown on the small-scale photographs of Barnard and Wolf and on plates taken with the 10-inch Cooke lens at Mount Wilson, but these photographs disclose only a little of the interesting detail. The subject of the present study is the brightest portion of this nebulosity, which is situated nearly halfway between a Cygni and the "isthmus" of N.G.C. 7000. It is shown on the Cooke plate reproduced in my third paper,¹ 30 mm from the right border and 64 mm from the bottom of the illustration.

The character of the nebulosity is much the same as that of the ¹ Mt. Wilson Contr., No. 256, Plate X; Astrophysical Journal, 57, Plate XIV, 1923.

southern part of the America nebula proper, consisting of luminous nebulosity in streaks and filaments with numerous dark markings. It is well shown in Plate VII, which is a good reproduction. The brightest star in the illustration is H.D. 199178, visual magnitude 7.56, situated 19 mm from the left edge and 69 mm from the top. In the lower right corner is the glow of light from 56 Cygni, magnitude 5.5. The following features are perhaps especially worthy of notice:

1. The slender dark marking, 2'.5 long, with a faintly luminous core and with a faint star near the end, which extends into the bright nebula eastward from a dark gulf just about the middle of Plate VII.

2. A similar, but shorter, dark indentation in the brightest part of the nebula, about 4' south of the first. Below this are two still smaller dark bays.

3. The long, parallel streaks of luminous nebulosity extending in a SE.-NW. direction across the upper left quarter of the plate.

4. The patch of faint stars extending westward from H.D. 199178.

5. The well-marked line of division between the bright and dark nebulosity that extends southward from the above-mentioned star.

6. The irregularly curved, bright filaments at the south end of the brightest part of the nebula.

The form of the luminous nebulosity has been likened to that of a pelican, the body of the bird being in the brightest part, the head rising nearly to the top of the plate, and the long, open beak extending nearly to the left edge.

Three spectrograms of the star H.D. 199178 are among those made with the spectrograph of the 60-inch telescope. The type is G5, as given in the *Henry Draper Catalogue*, but with unusually diffuse lines. Mr. Adams has kindly told me the result of his determination of the absolute magnitude of the star from these spectrograms, which is 3.5. With the apparent visual magnitude of 7.56 given in the *Henry Draper Catalogue*, this indicates a parallax of about 0".015, corresponding to a distance of 220 light-years. Whether or not this star is associated with the nebula is of course doubtful, but its location at an abrupt turn in the edge of the luminous nebulosity is suggestive.





PLATE VIII

M

Photographed with the roo-inch Hooker telescope. 1925, August 22 Exposure, $4^{\rm h}$ Scale: 1 mm = 15%8 (same as that of original negative)

PHOTOGRAPHIC STUDIES OF NEBULAE

Faint Nebula Following y Cygni

 $a = 20^{h}25^{m}5, \delta = +39^{\circ}46' (1925)$

Negative Δ 273, 60-inch telescope, 1925, August 20. Eastman Speedway plate, exposure 2^h

 Δ 278, Hooker telescope, 1925, August 22. Eastman Speedway plate, exposure 4^h. Illustrated in Plate VIII

This nebula follows γ Cygni by 6 minutes of time. It is shown 53 mm from the left edge and 82 mm from the bottom of the reproduction of Barnard's photograph made in 1894 with the Willard lens.¹ As shown in Plate VIII, it consists of a mottling of luminous and obscure nebulosities, a conspicuous feature being the large dark marking that extends southwestward from the prominent star, which is 43 mm from the left edge and 37 mm from the bottom of the plate. This star is B.D. +30°4206 = H.D. 195255, visual magnitude 8.80. Somewhat separated from the main mass of nebulosity, 15 mm from the right edge and 43 mm from the top of Plate VIII, is a nebulous star of the tenth magnitude, the nebulous character of which I believe has never before been recorded. Although not prominent on the photograph, the most conspicuous object seen by the eve in this field is the star 30 mm from the left edge and 44 mm from the bottom of the illustration, which in the field of the great reflector shone out startlingly with a deep red light. It was later identified as the variable star RW Cygni.

On September 1, 1925, I secured a spectrogram of the nebulous star above referred to, and one of H.D. 195255, with the Cassegrain spectrograph of the Hooker telescope, using the rapid camera of 10 inches focal length. Both spectra are of late B type. Mr. Joy, who examined them carefully, considers the brighter star to be of type B9 and the nebulous star to be somewhat earlier, perhaps B8.

Faint Nebula North-Preceding y Cygni

 $a = 20^{h} 14^{m} 1$, $\delta = +41^{\circ} 38' (1925)$

Negative Δ 281, Hooker telescope, 1925, August 23. Eastman Speedway plate, exposure 4^h. Images elongated. Illustrated in Plate IX

This nebula is 1.'6 north of γ Cygni, and precedes that star by 5 minutes of time. It is shown 81 mm from the right edge and 94 mm

¹ Publications of the Lick Observatory, 11, Plate 76, 1913.

JOHN CHARLES DUNCAN

from the top of the reproduction of Barnard's photograph of 1894.¹ It is quite similar in general appearance to the nebula shown in Plate VIII. A nebulous star appears 14 mm from the left edge and 45 mm from the top of Plate IX. This is considerably fainter than the one on Plate VIII, and a spectrogram has not yet been made. The brightest star appearing on Plate IX is B.D. $+41^{\circ}3693$, magnitude 8.6, which is 10 mm from the left edge and 20 mm from the top.

MOUNT WILSON OBSERVATORY AND WHITIN OBSERVATORY September 1925

1 Loc. cit



PLATE IX

Photographed with the 100-inch Hooker telescope, 1925, August 23 Exposure, $4^{\rm b}$ Scale: x mm = $x \xi^{*}_{*7}$ (1.02 times that of original negative)



INTERFEROMETER MEASUREMENTS OF THE PRESSURE-SHIFT OF LINES IN THE ARC SPECTRUM OF NICKEL¹

By B. T. BARNES

ABSTRACT

The wave-lengths for 72 lines in the nickel spectrum when emitted by an arc operated at a pressure of 76 cm of Hg are compared with those at 6 cm. Using a Fabry and Perot etalon, with a concave grating spectrograph for the auxiliary dispersive system, the nickel and the iron spectra were superposed and photographed simultaneously. The latter furnished standard lines from which to compute the nickel wave-lengths. By use of light from only the central region of a 12-mm arc, errors due to pole-effect were eliminated.

Most of the lines measured have a pressure-shift of less than 0.005 A per atmosphere. A few of the strongest lines showed an apparent shift of about 0.03 A per atmosphere when a 11-mm etalon was used. On changing to a 6-mm etalon, the shift measured was much less. Hence it is attributed to an unsymmetrical broadening of the line. This anomaly was found only in the case of very diffuse lines difficult to measure at atmospheric pressure. For the sharper lines, measurements on different plates agreed quite well. The probable error in the wave-lengths in Table I is, in most cases, not over 0.001 A.

INTRODUCTION

A number of years ago Duffield² measured the pressure-shift for some of the lines in the arc spectrum of nickel by comparing the wave-length emitted by an arc operated at a pressure of one atmosphere with that given by one at ten or more atmospheres. Since the change in wave-length is not directly proportional to the increase of pressure, his data do not furnish the information desired by spectroscopists and astrophysicists nowadays, namely, the difference between the wave-lengths obtained from a vacuum and from an atmospheric arc. Furthermore, the short arcs used at that time gave erroneous values for the pressure-shift because of pole-effect. This source of error was eliminated in the present investigation and the shift caused by altering the pressure from about 6 to 76 cm of mercury was measured directly.

APPARATUS

A cross-sectional view of the interferometer is shown in Figure 1. Three levers E, set opposite the screws in the ring C, press the lightly silvered glass plates A_1A_2 against the separator B. The

¹ Condensed from a Ph.D. dissertation presented at Yale University.

^{*} Philosophical Transactions, Royal Society, A, 215, 205, 1915.

B. T. BARNES

latter consists of three quartz pegs set in a brass ring. To allow separators of different lengths to be used, the inside of the tube D is threaded so that the ring C can be screwed into any position desired. This feature was copied from the interferometer designed by the Bureau of Standards.^I In order to make the tarnishing of the



FIG. 1.—The interferometer

silver less rapid the ends of the interferometer tube are closed by the plane-parallel glass windows F_1F_2 , and a drying tube G is attached, the Bureau of Standards having found² that silver surfaces retain their reflecting power when kept in a desiccator.

The auxiliary dispersive system consisted of a stigmatic grating spectrograph similar to that described by Meggers and Burns.³ Both the grating and the concave speculum mirror have a radius of curvature of about 640 cm. The grating is an excellent one, ruled by Anderson, with about 15,000 lines per inch and hence gives a dispersion around 5.1 A

per mm in the first order. This spectrograph differs from the one at the Bureau of Standards in that the I-beam along which the camera is moved from one part of the spectrum to another is in two sections, one of length 7 feet, and the other of 10 feet, at an

¹ Bulletin of the Bureau of Standards, 14, 712, 1918–19. ² Ibid., 7, 221, 1911. ³ Ibid., 18, 192, 1922–23.

PRESSURE-SHIFT OF NICKEL

angle of about 170°. The focal curve of the grating follows this beam much more closely than it would a single straight one.

As sources of light a nickel and an iron arc, each operated on a 220-volt circuit with 5 amperes current, were used. To make operation at low pressure possible the nickel arc was inclosed in a doublewalled cylindrical tube, with water circulating between the walls to keep the housing cool. Simultaneous exposure of the nickel and of the iron spectrum was obtained in the following way. An image of the nickel arc was formed in the iron arc by a short lens so placed as to give a magnification of unity. Then the light from both sources passed through a lens which focused it on a diaphragm placed in front of the nearer of the silvered plates of the interferometer; and the ring system produced by the light passing through this instrument was focused on the slit of the spectrograph by an achromatic lens of 25 cm focal length.

Both arcs were operated with an arc length of from 12 to 15 mm. Also the aperture in the diaphragm in front of the interferometer plates allowed the light from a region only $1\frac{1}{4}$ mm long in the center of each arc to pass through the etalon. Hence the specifications for the elimination of pole-effect adopted by the American Section of the International Astronomical Union¹ were followed in every detail.

RESULTS AND CONCLUSIONS

The wave-lengths given in Table I were computed by the method given by St. John and Babcock,² the values used for the standard Fe wave-lengths were those of Meggers, Kiess, and Burns.³ Every spectrogram was measured twice, the cross-hair traversing each line in opposite directions in the two cases, and usually the diameter of each of the first three rings was measured. Since each ring gives an independent value of the wave-length, the final result is the mean of three values.

In the first column of Table I is given the integral part of the wave-length, expressed in angstrom units. The next column contains the fractional part, as measured in the spectrum of the atmos-

¹ Proceedings of the National Academy of Sciences, 6, 307, 1920.

² Astrophysical Journal, 53, 263, 1921.

³ Bulletin of the Bureau of Standards, 19, 269, 1923.

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TABLE I

PRESSURE-SHIFTS IN THE NICKEL SPECTRUM

	Ат	MOSPHERIC	ARC		VACUUM ARC		
PART OF A	Decimal Part of λ	No. of Plates	Probable Error	$\begin{array}{c} Decimal \\ Part of \lambda \end{array}$	No. of Plates	Probable Error	PRESSURE- SHIFT
2406	0.354	т	h	0.255	2	b	-0.007
3527	.000	T	h	0.355	7	b	-0.001
07	.736	2	b	.725	T	6	1001
3610	.408	2	b	.500	2	6	- 002
61	.948	3	b	.946	2	b	.002
3664	.005	2	b	.002	2	b	002
74	.156	I	c	.140	T	b	007
88	.414	2	b	.413	2	h	.007
3730	.226	2	b	.227	2	b	- 001
72	. 529	2	b	.529	I	b	.000
3792	.343	2	a	.345	2	Ь	002
3831	.694	2	a	.692	2	b	.002
89	.674	2	C	.665	I	b	.000
3972	.343	2	a	.345	2	b	002
73	.559	2	b	.556	2	b	.003
3995	.310	2	b	.309	I	c	.00I
4284	.682	2	b	.679	I	C	.003
88	.00I	3	b	991	2	b .	.010
98	. 523	2	6	. 522	2	b	100.
4331	.647	4	b	.646	3	b	100.
4359	. 583	4	b	. 580	2	в	.003
4401	- 547	3	a	.541	2	a	.006
36	.982	3	b	.978	2	b	.004
59	.038	2	6	.035	2	b	.003
70	.483	2	C	.479	2	b	.004
4546	.935	2	C	.933	2	b	.002
92	.537	2	0	.530	2	C	.007
4000	.300	2	0	.301	2	b	.005
04	.993	2	0	.989	2	6	.004
00	.228	2	0	. 223	2	D	.005
4048	.050	2	6	.654	2	6	.002
4701	520	2	h	.215	2	6	.005
4/01	· 539	2	h	. 53/	2	0	.002
14	.425	2	c	.418	3	b	.003
4731	.803	I	c	.801	I	с	.002
32	.463	I	b	.458	I	C	.005
52	.423	2	C	.421	2	Ь	.002
54	.768	2	b	. 766	I	с	.002
56	. 520	2	b	.518	2	b	.002
4762	.632	2	b	.633	2	b	001
03	.950	2	b	.948	2	6	.002
80	.542	2	C	- 534	2	b	.008
4800	.993	3	0	.988	2	C	.005
29	0.032	3	0	0.025	2	0	0.007
PRESSURE-SHIFT OF NICKEL

Integral Part of a	Atmospheric Arc			VACUUM ARC			
	Decimal Part of λ	No. of Plates	Probable Error	Decimal Part of λ	No. of Plates	Probable Error	PRESSURE- SHIPT
4831	0.181	3	Ь	0.178	2	b	0.003
38	.651	3	b	.645	2	b	.006
52	. 565	3	b	. 556	2	C	.000
55	.418	3	a	.414	2	b	.004
57	.400	2	Ь	.395	2	C	.005
4866	. 277	3	a	.271	2	b	.006
73	.449	3	b	.446	2	b	.003
4004	.416	3	b	.414	2	b	.002
12	.024	3	b	.022	2	C	.002
13	.977	3	a	.975	2	C	.002
4918	.367	2	a	. 366	2	b	.001
25	. 569	3	b	. 568	2	b	100.
35	.836	3	a	.832	2	b	.004
53	.215	3	a	.200	2	b	.006
80	. 176	3	b	.176	2	b	.000
4984	.120	3	a	.115	3	c	.005
5000	.346	3	b	.344	2	C	.002
17	. 583	3	b	.579	2	b	.004
35	.365	3	b	.365	2	b	.000
42	. 196	2	b	. 202	I	C	006
5080	.537	3	b	.537	2	Ь	.000
81	.115	3	b	.110	3	C	.005
84	.095	3	G	.098	2	b	003
5115	.400	3	a	.397	2	b	.003
46	.485	3	C	.486	3	Ь	001
5155	.766	2	c	. 766	I	c	.000
68	0.664	2	6	0.665	2	6	-0.001

TABLE I-Continued

pheric arc, while the fifth column gives the fractional part of λ for the arc operated at a pressure of 6 cm of mercury. Usually each value is the mean of measurements made on two or three plates. The third and sixth columns give the number of plates measured in each case. In the fourth and seventh columns, the letters *a*, *b*, and *c* represent estimates, of less than 0.0005 A, of 0.0005 to 0.001 A; and of 0.001 and 0.002 A, respectively. Lines which could not be measured to this accuracy are not listed.

Between 3500 and 5200 A the pressure-shift was measured for most of the strong nickel lines. A few were too close to lines in the iron spectrum and a number of the most intense ones were too broad

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to give satisfactory fringes. Some of these strong lines, among them $\lambda\lambda$ 3548, 3612, 3775, 3783, 3807, and 3858, seemed to show a shift of about 0.03 A, when measured with a 12-mm etalon; with a 6-mm etalon, the shift was less in most cases. Probably unsymmetrical broadening caused this apparent shift.

All the lines recorded in Table I gave consistent results, since they are, in most cases, much sharper lines than the ones giving the abnormally large apparent shift just mentioned. Usually the shift is quite small; yet a few of the lines show a change in wave-length of about 0.01 A. It will be noticed that most of the pressure-shifts are positive and that none of the negative values are large. In some of the cases where the wave-length for the atmospheric arc is 0.001 A less than for the vacuum arc, this negative shift may be due to inaccuracy in the wave-length. However, negative shifts have been found for some lines in the spectra of other metals, and hence it is not surprising to find them in the nickel spectrum.

The wave-lengths in Table I are much more reliable than any previously obtained for the nickel spectrum. Most of them are accurate to 0.001 A. To be sure, the lines chosen for measurement are sharp ones, easily measurable with an ordinary grating spectrograph. The broad lines give unsatisfactory fringes. Yet the order of accuracy is greater than that usually obtained with grating spectrograms, and hence this measurement of pressure-shifts also yields valuable data as to wave-length.

In conclusion I wish to express my thanks to Professor Uhler, under whom this problem was carried out, for his aid in installing the optical system and for many valuable suggestions; to Professor Swann for his advice and assistance in connection with the sputtering of the interferometer plates, and to the other members of the Department of Physics for their interest and encouragement.

YALE UNIVERSITY NEW HAVEN, CONN.

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AN ATTEMPT TO DETECT WATER-VAPOR AND OXYGEN LINES IN THE SPECTRUM OF MARS WITH THE REGISTERING MICROPHOTOMETER¹

BY WALTER S. ADAMS AND CHARLES E. ST. JOHN

ABSTRACT

Observation of water-vapor and oxygen in the spectra of Mars and the sky.—If these gases are present in the atmosphere of Mars, the motion of the planet with reference to the earth should produce a relative displacement of the lines due to these gases in the two spectra.

Material.—Spectrograms of Mars and the sky were made with a 6-prism spectrograph, the scale being 1 mm=7.3 A at D; graphs were drawn by a registering microphotometer on a scale of 1 mm=0.12 A.

Method of measurement.—The lines of water-vapor and oxygen in the graphs of the two spectra were superposed; the relative displacements of the solar lines were then read directly from the curves.

Results.—The water-vapor lines in the spectrum of Mars were displaced 0.03 ± 0.01 A, and the oxygen lines 0.09 ± 0.03 A to the red, with respect to their positions in the sky spectrum. On taking account of the Doppler displacement, the length of path in the two atmospheres, and the amounts of water-vapor and oxygen above Mount Wilson, the quantity of water-vapor in the atmosphere of Mars, area for area, was found to be approximately 3 per cent of that over Pasadena, and the quantity of oxygen two-thirds of that above Mount Everest.

Before red-sensitive photographic plates were available, observations of the oxygen and water-vapor lines in the spectrum of Mars were necessarily visual. The first spectrograms covering the B-band of oxygen and the *a*-band of water-vapor² were made by V. M. Slipher in 1908.

Professor Very made photometric measurements of the relative intensity of the bands on Slipher's spectrograms and concluded that the atmosphere of Mars contained 1.75 times as much water-vapor as the atmosphere of the earth above Flagstaff and that the *B*-band of oxygen was 15 per cent stronger than in the spectrum of the moon at the same altitude.³

Campbell, from the equality of the *a*-band in spectrograms of Mars and the moon taken on Mount Whitney, came to the conclusion that the quantity of water-vapor in the atmosphere of Mars

¹ Contributions from the Mount Wilson Observatory, No. 307.

² Astrophysical Journal, 28, 397, 1908.

Lowell Observatory Bulletins, Nos. 36 and 41, 1909.

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at the time of his observations was too small to be detected by the spectroscopic methods then available.¹

The Doppler-Fizeau principle was applied by Lowell and Slipher with low dispersion in an attempt to detect lines originating in the atmosphere of Mars from their displacement with respect to similar lines originating in the atmosphere of the earth. They concluded that their measures neither proved nor disproved a displacement due to the motion of Mars relative to the earth.²

Campbell and Albrecht employed much higher dispersion and found for water-vapor lines near D a displacement of 19.2 km/sec. and for the α -band of oxygen 18.1 km/sec., the displacement due to relative motion being 19.1 km/sec. They concluded that the water-vapor in the atmosphere of Mars was less than one-fifth that above Mount Hamilton and that the quantity of oxygen was small compared to that in the earth's atmosphere.³

The primary purpose of our observations was to test the applicability of the Doppler-Fizeau principle to this problem by the use of a spectrograph of higher dispersion and the registering microphotometer. The spectrograms of Mars and of the sky were taken with a 6-prism wooden spectrograph placed on a pier in the constant temperature room at the coudé focus of the 60-inch reflector. The scale of the original spectrograms was 1 mm = 7.3 A at D and 11.4 A at λ 6300. The scale of the graphs drawn by the registering microphotometer was 1 mm = 0.12 A at D and 0.19 A at λ 6300. The spectrograms were made on February 2, 1925, at a time when the relative velocity of Mars and the earth was considerable.

Mars	Exp. 225 min.
Sky	Exp. 30 sec.
Slit width	0.075 mm
Length of path in atmosphere of Mars (sec z).	2.2
Length of path in atmosphere of Earth (sec z).	I.4
Relative velocity	+17.80 km/sec.
Velocity displacement at D	0.35 A
Velocity displacement at λ 6300	0.37 A

¹ Lick Observatory Bulletins, No. 169, 1909.

² Lowell Observatory Bulletins, No. 17, 1905.

3 Lick Observatory Bulletins, No. 180, 1909.

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Water-vapor.—The measurements were made by superposing successively the 6 water-vapor lines

λ 5879.820 5879.945	λ 5886.193	λ 5887.500 5887.620 5887.905	
$\left. \begin{array}{c} \lambda 5887.445 \\ 5887.880 \end{array} \right\}$	λ 5919.860	λ 5920.776	

in graphs of the spectra of Mars and the sky over millimeter crosssection paper and reading with a magnifier the distances between the lines of solar origin, $\lambda\lambda$ 5884, 5893, 5905, 5914, and 5922, on the two graphs. The means of the two observers were 2.6±0.13 mm and 2.7±0.10 mm, respectively, the final weighted mean being 2.67±0.08 mm, equivalent to 0.32±0.01 A.

The displacement of 0.35 A, due to relative motion, *minus* the measured displacement 0.32 A, is 0.03 A and represents the displacement to the red of the water-vapor lines in the combined Mars-sky spectrum due to water-vapor in the atmosphere of Mars. When the displacement is small compared with the width of the lines, as was the case on these graphs, the approximate relation

$\Delta \lambda = (\mathbf{I} + K_{\mathbf{I}}/K_2)\delta$

is applicable, where K_1 and K_2 are the intensities of the lines in the atmospheres of the earth and Mars, respectively. When $\Delta\lambda$ is 0.35 A and δ is 0.03 A, K_2 is 0.09 K_1 .

The component of the water-vapor lines due to Mars in the Mars-sky spectrum at the time of observation was 9 per cent as strong as the component due to the water-vapor in the earth's atmosphere. The precipitable water above Mount Wilson at 5 P.M. on the day of observation was 0.73 cu. cm, for the preceding day 0.74 cu. cm, and for the following 0.75 cu. cm, or one-half that over Pasadena at noon for the same days. Taking into consideration the paths traversed in the two atmospheres, we find that the quantity of water-vapor in the atmosphere of Mars, area for area, was 6 per cent of that over Mount Wilson and 3 per cent of that over Pasadena. This indicates extreme desert conditions over the greater portion of the Martian hemisphere toward us at the time, which was

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near the beginning of the Martian spring, or about 2.5 Martian months after the solstice.

Oxygen.—Similar measurements were made with the oxygen lines.

λ 6276.815	λ 6277.513	λ 6279.084	λ 6295.389
6277.021 5	6277.634	6279.308 5	6296.170
	6277.701		6310.101
	6277.837)		

The mean displacement of the solar lines $\lambda\lambda$ 6265, 6270, 6298, 6301-2, 6322, and 6339, referred to the oxygen lines in the Mars-sky



FIG. I

spectrum, was 1.46 ± 0.17 mm, equivalent to 0.28 ± 0.03 A. The displacement due to relative velocity was 0.37 A. Applying the formula as above, we find that the Martian component of the oxygen line in the Mars-sky spectrum was 33 per cent of the earth's component.

On taking account of length of path and the elevation of the observing station, the oxygen in the atmosphere of Mars was found to be 16 per cent, area for area, of that over Mount Wilson, or about two-thirds of that in the earth's atmosphere above Mount Everest.

The heights of the curves M and E in Figure 1 indicate the relative intensities of the Martian and terrestrial components of the

SPECTRUM OF MARS

oxygen and water-vapor lines on the Mars-sky spectrogram. The Martian components are displaced to the red—to the left in the diagram—by the amount of the Doppler effect. For components of equal intensity the compound line in the spectrum of Mars would have been displaced by one-half the Doppler effect. The measured displacement, which is much less, is indicated by the position of the solid vertical line.

We wish to express our appreciation of the skill and experience of Miss Ware in the production, and her assistance in the measurements, of the microphotometer graphs.

MOUNT WILSON OBSERVATORY November 1925

REVIEWS

Théorie mathématique de l'Électricité. By TH. DE DONDER. Première Partie: Introduction aux Equations de Maxwell, edited by G. VAN LERBERGHE. Paris: Gauthier Villars, 1925. 4vo. Pp. 200. Fr. 45+10 per cent postage.

This book is the first part of the *Mathematical Theory of Electricity*, by M. De Donder, professor at the University of Brussels. It is a systematic treatment of the subject, enabling the reader to obtain Maxwell's equations in the greatest generality. Applications of the Maxwell-Lorentz equations and the electronic conception of electric structure are promised for a second volume.

The work is divided into three parts, treating successively of the electrostatic field, the stationary electromagnetic field, and the variable electro-magnetic field.

The first subdivision includes eight chapters, dealing successively with the point distribution of electricity, the electric volume distribution, the continuous surface distribution with applications to perfect conductors, the electric bipolar volume distribution with its application to perfect homogeneous and isotropic dielectrics, and the bipolar surface distribution.

The four chapters which form the second part, the stationary electromagnetic field, are devoted to the volume and surface distribution of currents with application to perfect conductors, the magnetic bipolar volume and surface distribution, and finally to some general theorems on the stationary electromagnetic field. Of special interest are the distinction made between the electric (or magnetic) force and the resultant, the several aspects of the energy of a bipolar distribution, the treatment of the ellipsoidal hollow conductor, and the distribution of surface and volume currents.

In the third part, which involves the study of the variable electromagnetic field, Maxwell's equations are enunciated under the most general form. These equations lead to the consideration of the electromagnetic potentials and to the differential equations which they satisfy. The theorem of the conservation of energy is given with entire generality. From Maxwell's stresses are deduced the mechanical force and the couple

REVIEWS

per unit volume. Special attention is given to some important particular cases and to the virtual work due to the field.

The volume ends with a complete study of all the systems of units used in the application of electricity; and the synoptic table of the dimensions will prove to be of value to all students who are interested in practical investigations.

For reading this work, only elementary notions of infinitesimal calculus are needed, and no preliminary knowledge of vectorial calculus is required.

The subject is considered in a simple, natural, and logical way. By reason of its generality, the book may be regarded as a treatise on electricity.

H. L. VANDERLINDEN

College Physics. By A. WILMER DUFF. New York: Longmans, Green & Co., 1925. 8vo. Pp. xii+484. Figs. 499. Price, \$3.80.

This textbook covers almost the whole field of physics in a clear yet concise manner. The author has endeavored "to simplify and abbreviate the mathematical work without making it less rigorous." The result is interesting. To the student who desires a general knowledge of physics as an auxiliary to his main line of study, this book robs the subject of the much-dreaded mathematical reasoning, replacing such by clear logical statements leading directly to the formulae required. On the other hand, the student aiming toward advanced work in physics would be apt to get a very inadequate conception of what the physicist regards as a rigorous treatment.

Undoubtedly those who teach physics to the first type of student will find this book a very valuable addition. The frontispiece is admirable, showing as it does in diagrammatic form the gradual development of physics since 600 B.C. by the cumulative contributions of the great natural philosophers of twenty-five centuries.

Of the 470 pages, the first 100 are devoted to mechanics. A brief treatment of the properties of liquids and gases follows with a good introduction to wave-motion. Sound is accorded twenty pages. An excellent treatment of heat is given in 75 pages, bringing the reader to the idea of entropy. Electricity and magnetism are carefully covered in about 130 pages, and include a brief discussion of wireless telegraphy, cathode rays, X-rays, radioactivity, and modern atomic theory. The last section is devoted to light, under the usual subdivisions with the addition at relevant points of paragraphs on relativity, crystal analysis, and quantum theory.

REVIEWS

The book is well supplied with examples of a simple, direct type and with questions calculated to encourage constructive thought on the part of the student. Many of the illustrations are essentially modern, as, for example, the brief explanation of Flettner's rotor ship following the derivation of Bernoulli's theorem. The desire has been to educate, not merely to teach the facts of physics; thus in treating earth's magnetism reference is made to the expeditions of Amundsen and Shackleton.

Twenty-seven tables of physical constants and properties are inserted in the text, and an Index of Tables is provided. A concise table of logarithms and selected trigonometrical functions is given at the end of an Appendix containing the derivation of a few important formulae.

On page 98 the word "revolve" should replace "rotate" in the enunciation of Kepler's law. On the following page the author attempts a difficult task. We think he would have succeeded better had he headed his paragraph in some such way as "Einstein's Modification of the Law of Gravitation." Is it legitimate to say that he has "explained" gravitation? His law appears to fit the facts of nature more precisely than does Newton's but surely that is as far as we are justified in going. A student starting hopefully from the heading of paragraph 112 may be enlightened as to the method of attack upon the problem but not as to the promised explanation of the phenomenon. We are certain the author could have done better, and even if he had filled another half-page it would have been worth while.

The omission of the name of Sommerfeld in the paragraph on Bohr's theory (p. 437) seems unjustifiable. One could wish that some of the outstanding problems of geophysics had been touched upon. Even the briefest references to such questions as isostacy and earthquake waves help the beginner to realize the ramifications of physics and therefore its fundamental place among the sciences.

But these are minor points; the recent advances in physics are so many that it is a triumph to have embraced so much of the subject in as compact a form as Dr. Duff has succeeded in doing.

A. VIBERT DOUGLAS

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